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KENDRIYA VIDYALAYA SANGATHAN

REGIONAL OFFICE AGRA

STUDENT STUDY MATERIAL

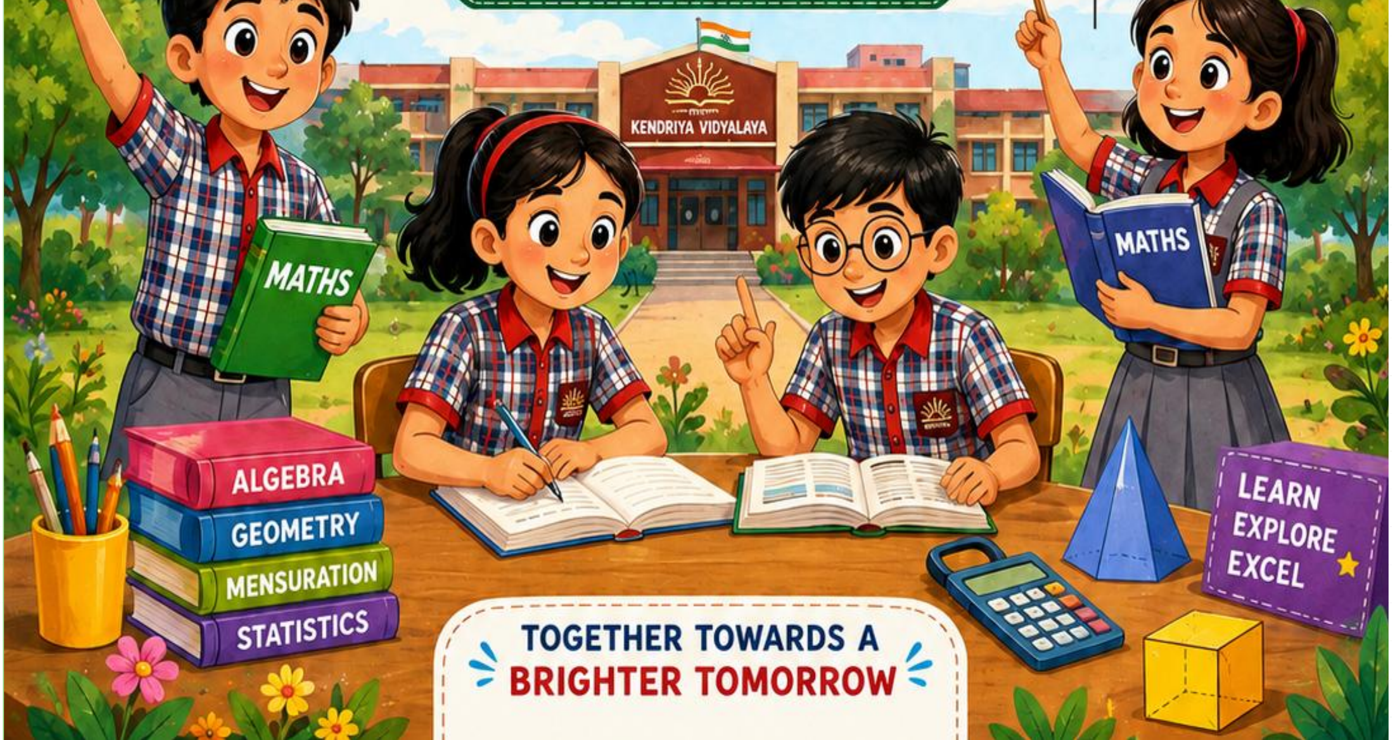
GANIT MANJARY

PART 1

(MATHEMATICS)

CLASS - 9

SESSION-: 2026-27



TOGETHER TOWARDS A BRIGHTER TOMORROW

MATHEMATICS STUDY MATERIAL

(SESSION 2026–27)

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We extend our heartfelt gratitude to our worthy patrons for their valuable guidance, constant encouragement and visionary leadership.

TEAM – MATHEMATICS

OUR TEAM

SL.NO.	Chapter (IX Mathematics)	Teacher Allotted (TGT Maths)	School Name
1	Orienting Yourself: The use of coordinates	Mrs. Geetanjali Rajpoot	PM SHRI No.3 Jhansi
2	Introduction to linear Polynomials	Mrs. Richa Mittal	PM SHRI KV Noida (Shift 1)
3	The world of numbers	Mr. Avdhesh Kumar Niranjan	PM SHRI KV Babina
4	Exploring Algebraic Identities	Mr. Shiva	KV NTPC Dadri
5	I'm Up and Down and Round and Round	Mr. Alok Kumar	PM SHRI KV PL Meerut
6	Measuring Space: Perimeter and Area	Mr. Devendra Kumar	PM SHRI KV SL Meerut
7	The Mathematics of Maybe: Introduction to Probability	Mrs. Pooja	PM SHRI KV Mahoba
8	Predicting What Comes Next: Exploring Sequences and Progressions	Mr. Hemraj	PM SHRI KV O.F. Muradnagar

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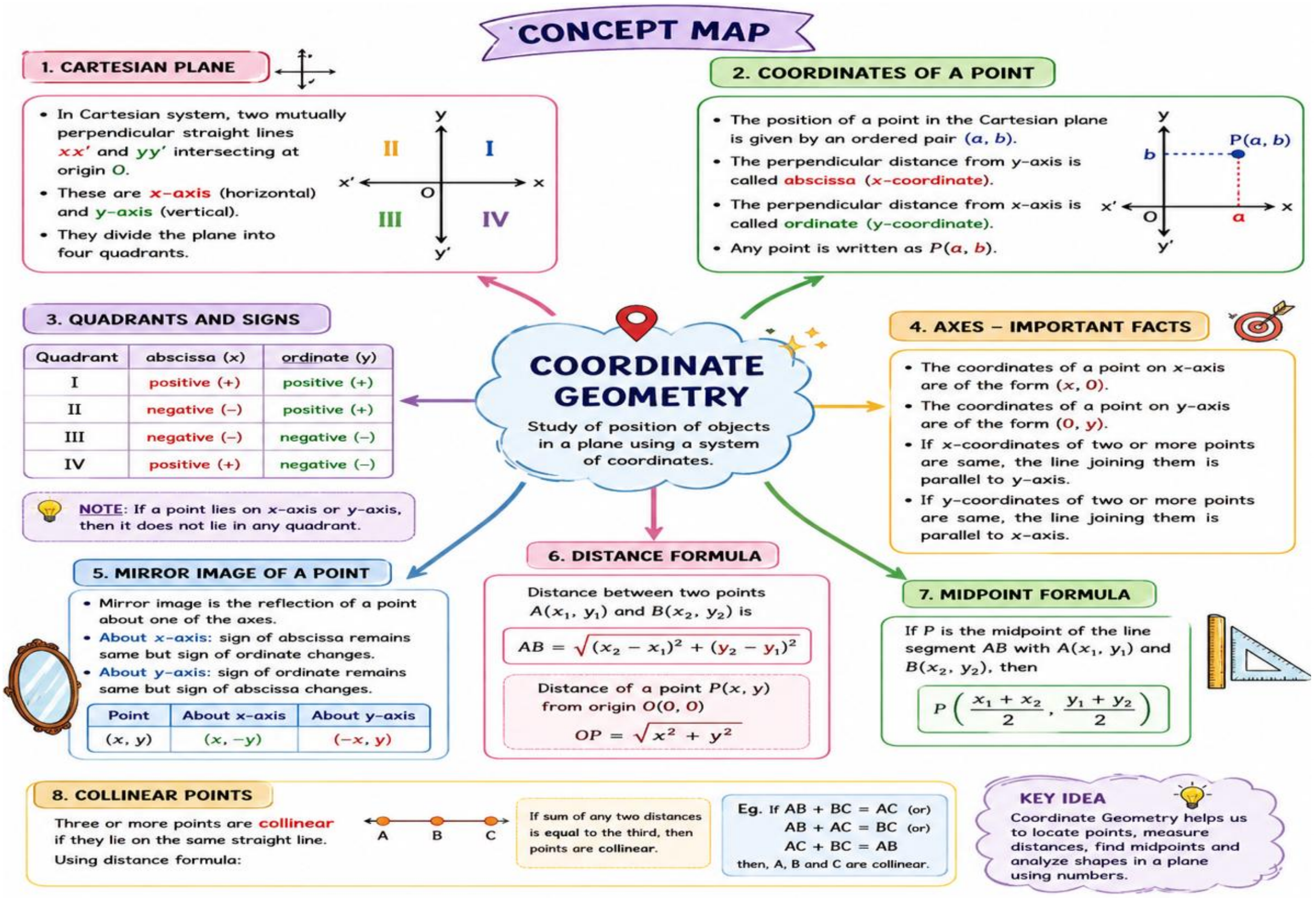
CHAPTER-1

Orienting Yourself: The Use of Coordinates



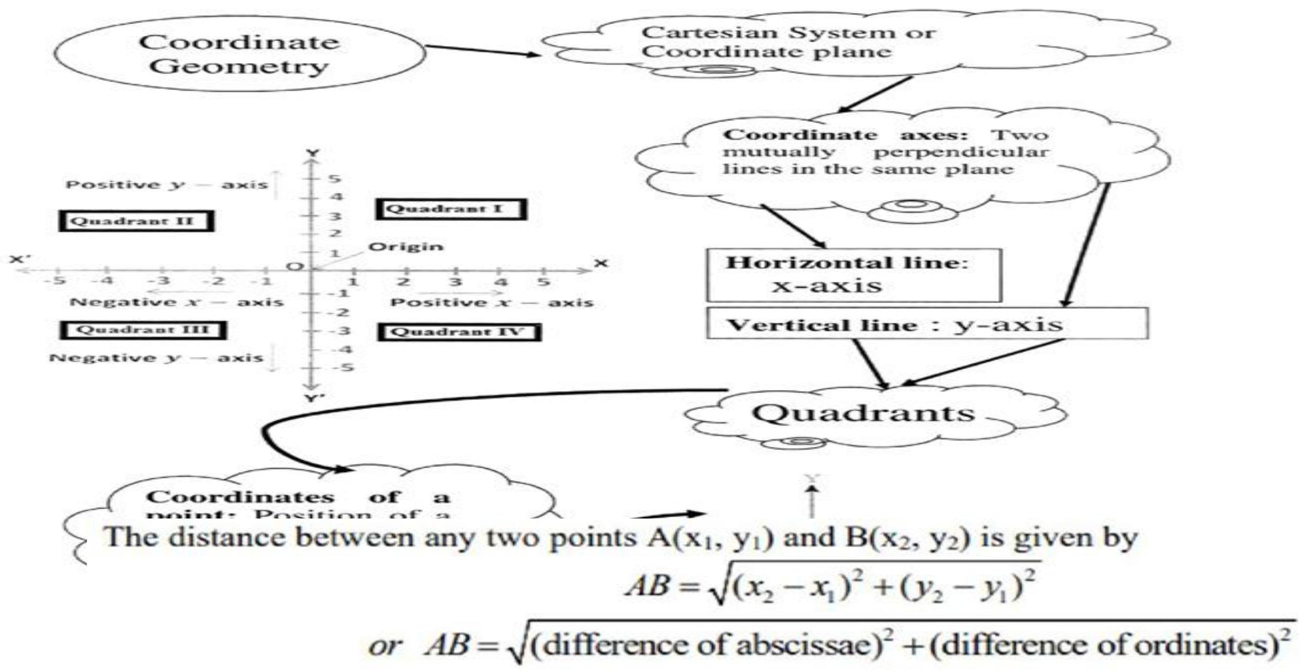
KEY POINTS:

- Co-ordinate Geometry is the branch of Mathematics in which we study the position of any object lying in a plane, called the Cartesian plane.
- In Cartesian system; there are two mutually perpendicular straight lines xx' and yy' intersecting at origin O.
- These mutually perpendicular straight lines, known as x-axis and y-axis,divides the plane into four quadrants.
- The coordinates of a point is the position of the point in Cartesian plane and are determined by perpendicular distance from x-axis and y-axis.
- The perpendicular distance of a point from y-axis is called abscissa (x-coordinate) and from x-axis is called ordinate (y-coordinate).
- Any point in the Cartesian plane is shown by P(a, b) where (a, b) are coordinates of point P.

abscissa (x)	**ordinate (y)**	**Position of point**
positive (+)	positive (+)	Quadrant I
positive (-)	negative (+)	Quadrant II
negative (–)	negative (–)	Quadrant III
positive (+)	negative (–)	Quadrant IV

• The coordinate of a point on x-axis is of the form (x, 0) and on y-axis is of the form (0, y).
• If x-coordinate of two or more points are same, then the line joining these points is parallel to y- axis.
• If y-coordinate of two or more points are same, then the line joining these points is parallel to x axis.
NOTE: If point lie on x-axis or y-axis then it does not lie in any quadrant.
• The **mirror image** of a point is just a reflection of this point about one of the axes. Mirror image about x-axis: sign of abscissa remains same but sign of ordinate changes.
• The distance between the points (x_1, y_1) & $(x_2, {}_2)$ = $\sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2\}}$
Note: If O is the origin, the distance of a point P(x, y) from the origin O(0, 0) is given by
OP = $\sqrt{(x^2 + y^2)}$
• **Mid-point Formula:** If P is the mid-point of line segment AB, the coordinates of P are:
$[\frac{x1+x2}{2}, \frac{y1+y2}{2}]$
• **COLLINEAR POINTS:** A given number of points are said to be collinear if they lie on the same line. To prove that three points A ,B and C are collinear (using distance formula), we need to prove that sum of any two of the distances AB, BC and AC is equal to the third distance.

MIND MAP



Q1. The point P on x-axis equidistant from the points A(– 1, 0) and B(5, 0) is?
Sol. Coordinates of A = (-1, 0)
Coordinates of B = (5, 0)

Let the coordinate of the required point be M(x, 0)

M is equidistant from A and B $\Rightarrow AM=BM$

$\sqrt{\{(-1-x)^2+(0-0)^2\}}=\sqrt{\{(5-x)^2+(0-0)^2\}}$

On squaring both sides, $(-1-x)^2=(5-x)^2$

$$1+x^2+2x=25+x^2-10x$$

$$1+2x=25-10x$$

$$12x=24 \Rightarrow x=2$$

Required point is (2, 0)

Q2. Find the coordinate of the point

(i) which lies on the x and y axes both.

(ii) whose ordinate is -5 and which lies on y - axis.

Sol. I) A point lying on the **x-axis** has y=0.

A point lying on the **y-axis** has x=0.

Therefore, a point lying on both axes is the **origin,** the coordinate of the origin(0,0).

ii) We know that: Ordinate = y-coordinateOn y-axis, x=0, Given ordinate=−5.Therefore, the coordinate of the point is (0, −5).

Q3. Find the point of bisect of the segment joining the points (3, -2) and (-3, 4).

Sol. The coordinate of mid point =[(3−3)/2, (-2+4)/2] = (0,1)

Q4. If the point P (x, y) is equidistant from the points A (a + b, b – a) and B (a – b, a + b).Prove that bx = ay.

Sol. Given: P(x,y) , A(a+b,b−a) and B(a−b,a+b)

Since P is equidistant from A and B, then PA=PB

Squaring both sides:$PA^2=PB^2$

Using the distance formula,

$$[x-(a-b)]^2+[y-(b+a)]^2=(x-a+b)^2+(y-b-a)^2$$

Expanding and simplifying :− 4ay+4bx=0

Therefore, 4bx=4ay

Hence, bx=ay **Hence proved.**

Q5. Points A(-1 ,y) and B(5, 7) lie on a circle with centre O(2 ,-3y). Find the values of y by using distance formula. Hence, find the radius of the circle.

Sol. Since A and B lie on the same circle with centre O,

Then, OA=OB

Finding OA^2 : A(−1,y),O(2,−3y) Using distance formula:

$OA^2=(2+1)^2+(-3y-y)^2=32+(-4y)^2=9+16y^2$

Finding OB^2: B(5,7),O(2,−3y),

$OB^2=(5-2)^2+(7+3y)^2=9+(7+3y)^2$Since OA=OB,

$9+16y^2=9+(7+3y)^2$

$16y^2=(7+3y)^2$

$16y^2=49+42y+9y^2$

$7y^2-42y-49=0$ (Dividing by 7)

$y^2-6y-7=0$ Factorising:

$(y-7)(y+1)=0$,Therefore, $y=7$ or $y=-1$

Finding the radius

Case 1: $y=7$, then $r^2 = OA^2 = 9+16(7)^2 = 9+784 = 793$
$r = \sqrt{793}$ unit

Case 2: $y=-1$, then $r^2 = OA^2 = 9+16(-1)^2 = 9+16 = 25$
$r = 5$ unit

Answer: If $y=7$ unit, $r=\sqrt{793}$ unit **or** $y=-1$ unit, $r=5$ unit

Q6. Prove that the point $(x, \sqrt{(1-x^2)})$ is at a distance of 1 unit from the origin.

Sol. $P=[x,\sqrt{(1-x^2)}]$ and the origin be $O=(0,0)$

Using the distance formula:
$OP=\sqrt{[(x-0)^2+\sqrt{\{(1-x^2)-0)\}^2}]} = \sqrt{(x^2+1-x^2)} = 1$
$OP=1$

Therefore, the point $[x,\sqrt{(1-x^2)}]$ is at a distance of 1 unit from the origin. **Hence proved.**

Q7. Prove that the points (3, 1), (6, 4) and (8,6) are collinear

Sol. Let A (3,1), B (6,4) and C (8,6)

We can use the **distance formula**.

Find AB, $AB=\sqrt{[(6-3)^2+(4-1)^2]} = \sqrt{(9+9)} = 3\sqrt{2}$ unit

Find BC, $BC=\sqrt{\{(8-6)^2+(6-4)^2\}} = \sqrt{(4+4)} = 2\sqrt{2}$ unit

Find AC, $AC=\sqrt{[(8-3)^2+(6-1)^2]} = \sqrt{(25+25)} = 5\sqrt{2}$ unit

Therefore, $AB+BC = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2} = AC$

Since AB+BC=AC, the point B lies between A and C.

Hence, the three points are **collinear**. **Hence proved**.

Q8. Ronit is the captain of his school football team. He has decided to use 4-4-2-1 formation in the next match. The above figure shows the position of the players in 4-4-2-1 formation on a coordinate grid

(i). What are the coordinates represents the position of the goalkeeper?

The goalkeeper is located on the **y-axis at y=−9**.

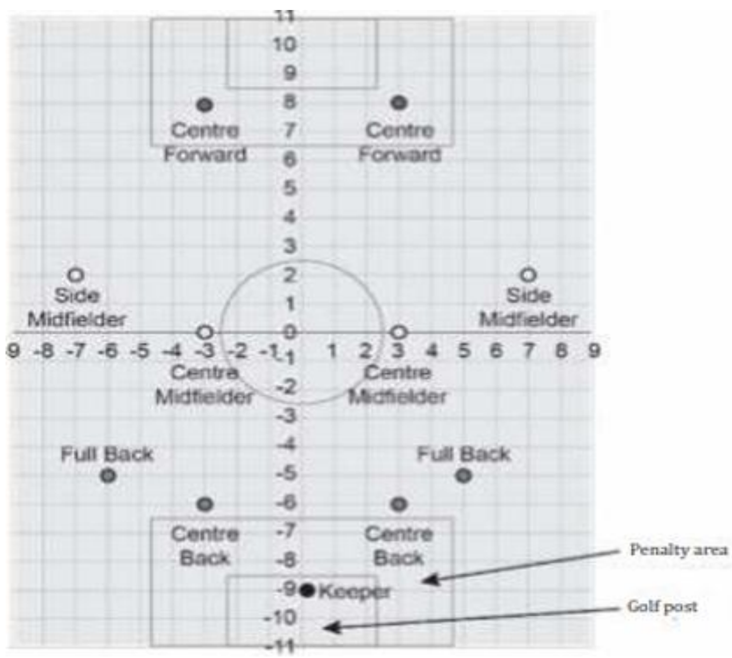
Answer: (0,−9)

(ii). What is the distance between the two centre forward positions in Ronit's plan?

The two centre forwards are at: $(-3,8)$ and $(3,8)$

Since they have the same y-coordinate,

the distance $= 3-(-3) = 6$ unit



Answer: 6 units

(iii). Mention two positions which are not equidistant from any axis.

For a point (x,y):

- Distance from the **x-axis** = |y|
- Distance from the **y-axis** = |x|

For example, the **Side Midfielder** at (−7,2) has distances 2 and 7, so they are not equal.

Similarly, the **Full Back** at (−7,−4) has distances 4 and 7.

Answer: Any two suitable positions are:

- (−7,2) — Side Midfielder
- (−7,−4) — Full Back

MULTIPLE CHOICE QUESTIONS

Q1. The section formed by horizontal & vertical lines determining the position of point in a Cartesian plane is called:
a) Origin b) X-axis c) Y-axis d)Quadrants

Q2. The point whose ordinate is 8 and lies on y-axis:
a) (0, 8) b) (8, 0) c) (5, 8) d) (8, 5)

Q3. If the coordinates of a point are (-3,-4), then it lies in:
a) First quadrant b) Second quadrant
c) Third quadrant d) Fourth quadrant

Q4. The mirror of a point (3, 4) on y-axis is:
a) (3, 4) b) (−3, 4) c) (3, −4) d) (−3, −4)

Q5. Signs of the abscissa and ordinate of a point in the second quadrant are respectively
a) +, + b) +, − c) -, + d) -, −

Q6. If x > 0 and y < 0 , the point (x , -y) lies in
a) I quadrant b) II quadrant c) III quadrant d) IV quadrant

Q7. If point P(3,0) and Q(a,0)are equidistant from the origin & a then a =
a) 3 b) - 3 c) 6 d) 9

Q8. Distance between two points (3, 2) and (6, 6) is:
a) 5 b) 3 c) 2 d) 8

Q9. Ordinate of all the points on the X-axis is
a) 0 b) 1 c) 2 d) Any number

Q10. The perpendicular distance of the point P (-5, 7) from the y-axis.
a) 5 b) -5 c) 7 d) -7

Q11. The co-ordinate of the point which is reflection of point (−3, 5) in X- axis are
a)(3, 5) b) (3, −5) c) (−3, −5) d) (−3, 5)

Q12. The distance of the point P (-6, 8) from the origin is:
a) 14 b) 6 c) 8 d) 10

Q13. If the distance between P(4, 0) and Q(0, x) is 5 units, the value of x will be

a) 2 b) 3 c) 4 d) 5

Q14. The point P on x-axis is equidistant from the points A(-1, 0) and B(5, 0) is:
a) (2, 2) b) (0, 2) c) (2, 0) d) (3, 2)

Q15. The distance of the point P(α, β) from the origin is:
a) $\alpha + \beta$ b) $\alpha^2 + \beta^2$ c) $|\alpha| + |\beta|$ d) $\sqrt{\alpha^2 + \beta^2}$

Q16. The abscissa of a point is the distance of the point from
a) origin b) x-axis c) y-axis d) None of these

Q17. The end points of a line lies in I quadrant and III quadrant. The line may pass through
a) origin b) negative x-axis c) positive y-axis d) quadrant II

Q18. Point (a, 0) lies
a) on x-axis b) on y-axis c) in third quadrant d) in fourth quadrant

Each of the following questions containing Assertion (A) and Reason (R) has the following choices:
a) If Assertion (A) is true, Reason (R) is true; Reason (R) is correct explanation for Assertion (A).
b) If Assertion (A) is true, Reason (R) is true; Reason (R) is not correct explanation for Assertion
c) If Assertion (A) is true, Reason (R) is false.
d) If Assertion (A) is false, Reason (R) is true.

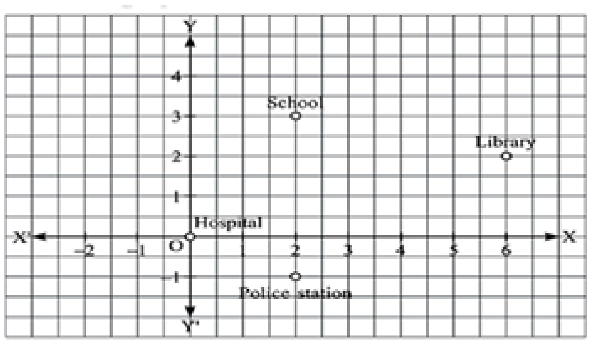
Q19.**Assertion (A)** : The point (0, 4) lies on y-axis.
Reason (R) : The x-coordinate of a point, lying on y-axis, is zero.

Q20.**Assertion (A)**:The distance of the point (-4,5) from the X-axis is 4 units.
Reason (R) : Abscissa of a point gives the distance from the Y-axis.

VERY Short Type Questions (2 marks each)

Q21. Find a point on y-axis which is equidistant from the point A (6, 5) and B (-4, 3).

Q22. Answer the following questions on the basis of graph given below:



i. What are the coordinates of School?
ii. What are the coordinates of Police Station?
iii. What are the coordinates of Hospital?
iv. What are the coordinates of library?

Q23. TRUE / FALSE

i. The origin is the only common point of the two axes.
ii. In a graph paper, the vertical axis is usually known as the x-axis.
iii. On a graph paper, the positions of the points (1,4), (4,1) coincide.
iv. The coordinate axes divide the plane into four parts.

Q24. .Without plotting the points indicate the quadrant in which they will lie, if
(i) the ordinate is 1 and abscissa is – 1
(ii) the abscissa is – 5 and ordinate is – 2

Q25. If the points (2, 1) and (1, -2) are equidistant from the point (x,y). Show that $x + 3y = 0$.

Short Type Questions (3 marks each)

Q26. On plotting the points O(0, 0) , A (3, 0) , B(3, 4), C (0, 4) and joining OA, AB, BC and CO. Name the figure obtained? Also find the length of its diagonal.

Q27. Taking 0.5 cm as 1 unit, plot the following points on the graph paper:

A(1,3) , B(-3,-1) , C(1, -4) ,D(-2,3) ,E(0,-8), F(1,0)

Q28. Three vertices of a rectangle are (3,2), (-4,2), and (-4,5). Plot these points and find the coordinates of the fourth vertex.

Q29. If the point (0, 2) is equidistant from the points (3, k) and (k, 5), find the value of k.

Q30. If the distance between the points (4, p) and (1,0) is 5, then what can be the possible values of p?

LONG ANSWER TYPE QUESTIONS (5 marks each)

Q31. Find the co-ordinates of the point which divides the line segment joining the points A(2, 6) and B(10, –10) in to 4 equal points.

Q32. If A(x, y), B(– 2, 3) and C(2, 1) form an isosceles triangle with AB = AC. Also find the relation between x and y.

Q33. Consider the points R(3, 0), A(0, −2), M(−5, −2) and P(−5, 2). If they are joined in the same order, predict:

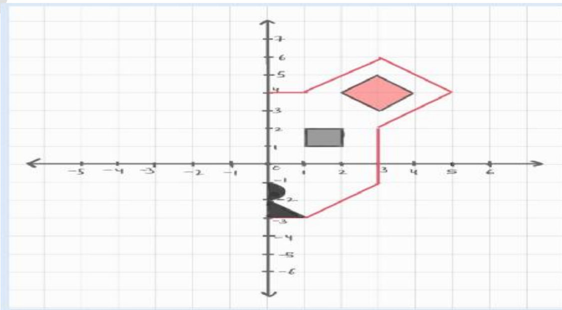
(i) Two sides of RAMP that are perpendicular to each other.

(ii) One side of RAMP that is parallel to one of the axes.

(iii) Two points that are mirror images of each other in one axis. Which axis will this be?

(iv) Now plot the points and verify your predictions.

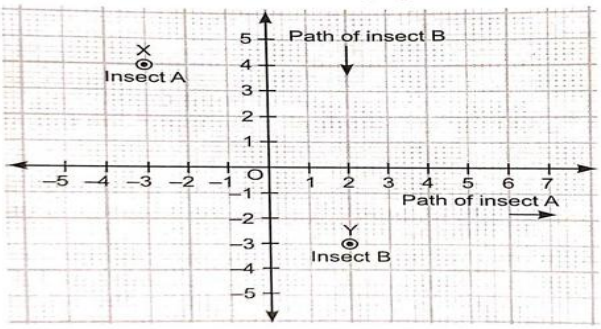
Q34. Rony was reading a Maths magazine in which he saw an interesting figure which was half drawn as shown below. He wish to draw it completely. Draw again and help him to complete the figure and tell which line of symmetry you use. Along with that mark all the coordinates used in this drawing.



Q35. Plot the points A (2, 1), B (−1, 2), C (−2, −1), and D (1, −2) in the coordinate plane. Is ABCD a square? Can you explain why? What is the area of this square?

<u>Case Based Study Question</u>

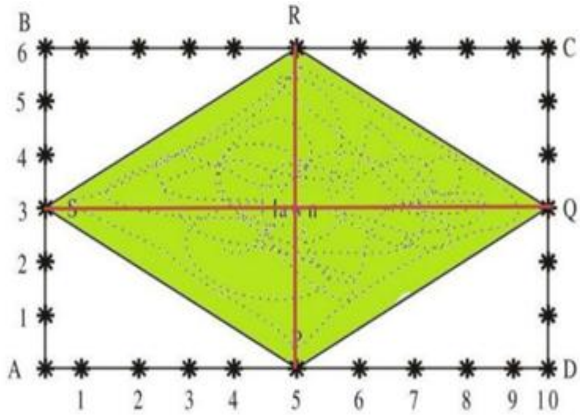
Q36. There are two insects A and B. A is moving from left to right horizontally and B is moving from top to bottom vertically. We have divided their path into unit distance of 1cm and it looks like a cartesian plane.



After tracing path, insect A is at point X and insect B is at point Y as shown in graph.

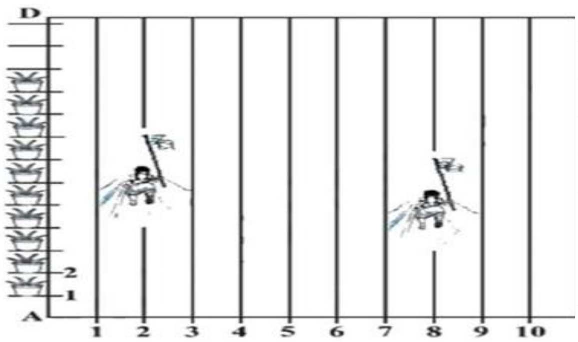
- **i.** The coordinate where abscissa is + and ordinate is - , lies in which quadrant.
- **ii.** What is the ordinate of any point on x-axis.
- **iii.** What is the distance between both insects?

Q37. The Class IX students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Sapling of Gulmohar are planted on the boundary at a distance of 1m from each other. There is a lawn PQRS in the ground as shown in figure



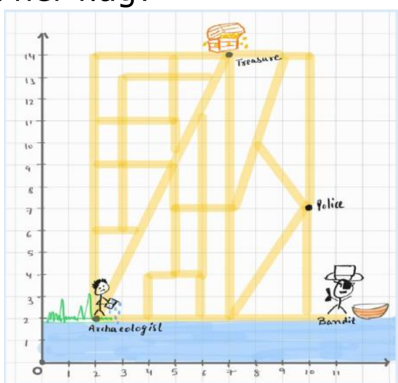
(a) The coordinates of C, taking A as origin?
(b) What are the coordinates of R, taking A as origin?
(c) What is the length of side of lawn?
OR
What is the area of lawn?

Q38. To conduct Sport Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in figure. Hema runs 1/4 th the distance AD in the 2nd line and posts a blue flag. Preeti runs 1/5 th the distance AD on the 8th line and posts a green flag.



a) Which mathematical concept is used in the above problem?
b) What is the position of blue flag and green flag?
c) What is the distance between both the flags? **OR**
If Uttara has to post an orange flag exactly halfway between the line segment joining the two flags, where should she post her flag?

Q39. Once an Archaeologist was doing research in cave where he got a map to find treasure. So he went in search of treasure. But when he reached there his map fell into the water and everything vanished from the map. On the other side Bandit know the way to reach there but Police is behind him to catch. (1 unit in map = 1Km).



i. To find the **coordinates of the treasure.**
ii. What are the coordinates of the police officer?
iii. Find the shortest distance between the archaeologist and the treasure.
OR
iv. Find the distance between the police officer and the bandit.

ANSWERS: -

1. d) Quadrants
2. a) (0, 8).
3. c) Third quadrant.
4. b) (−3, 4).
5. c) −, +.
6. d) IV quadrant
7. a= 3
8. a) 5
9. a) 0
10. a) 5
11. c) (−3, −5).
12. d) 10
13. b) 3
14. c) (2, 0).
15. d) $\sqrt{\alpha^2 + \beta^2}$
16. c) y-axis
17. a) origin
18. a) on x-axis
19. a) Assertion is true, Reason is true, and Reason correctly explains Assertion
20. d) Assertion is false, Reason is true.
21. P=(0,9).
22. Answer: i) (2,3) (ii) (2,-1) (iii) (0,0) (iv) (6,2)
23. (i) True. (ii) False — the vertical axis is the y-axis. (iii) False — (1,4) and (4,1) are different points. (iv) True.
24. (i) (−1,1) has signs (−,+), so Quadrant II. (ii) (−5,−2) has signs (−,−), so Quadrant III.
25. The source prints the point incompletely as "(x,)." Assuming the intended point is (x,y): $(x-2)^2+(y-1)^2=(x-1)^2+(y+2)^2$. On simplifying, $x+3y=0$.
26. OA=3, AB=4, BC=3 and CO=4. Therefore the figure is a rectangle. Its diagonal = $\sqrt{3^2+4^2}=5$ units
27. Plot A(1,3), B(−3,−1), C(1,−4), D(−2,3), E(0,−8) and F(1,0), using the stated scale of 0.5 cm = 1 unit.
28. The missing fourth vertex is (3,5), because the x-coordinates are −4 and 3 and the y-coordinates are 2 and 5.
29. $9+(k-2)^2=k^2+9$. Hence $(k-2)^2=k^2$, giving $-4k+4=0$, so $k=1$.
30. $\sqrt{(4-1)^2+p^2}=5 \Rightarrow 9+p^2=25 \Rightarrow p^2=16$. Therefore $p=\pm4$.

31. The three points dividing A(2,6) to B(10,−10) into four equal parts are (4,2), (6,−2) and (8,−6).
32. AB=AC gives $(x+2)^2+(y-3)^2=(x-2)^2+(y-1)^2$. Simplifying: $8x-4y+8=0$, hence $2x-y+2=0$, or $y=2x+2$.
33. (i) PM⊥AM and PR ⊥MP (ii) AM is parallel to the x-axis; MP is parallel to the y-axis. (iii) No points are mirror image
34. Reflected coordinates (left half, x → −x, y unchanged):

Outline: (0, 4), (−1, 4), (−3, 6), (−5, 4), (−3, 2), (−3, −1), (-1,-3), (0,-3)

Diamond: (−2, 4), (−3, 5), (−4, 4), (−3, 3)

Rectangle: (−1, 1), (−2, 1), (−2, 2), (−1, 2)
35. $AB^2=BC^2=CD^2=DA^2=10$, while $AC^2=BD^2=20$. Thus all four sides are equal and the diagonals are equal, so ABCD is a square. Area = side² = 10 square units.
36. i. Quadrant IV.
ii. The ordinate of any point on the x-axis is always 0.
iii. Distance = √75 unit
37. (a) Coordinates of C = (10, 6)
(b) Coordinates of R = (5, 6)
(c) Length of side of lawn PQRS:
Length of side lawn $= \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$ m ≈ 5.83 m OR
Area of Lawn $= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 10 \times 6 = 30$ m^2
38. a) Cartesian coordinate system (coordinate geometry)
b) Position of the flags: blue flag(2 , 25), green flag (8 , 20)
c) Distance between the two flags: √61 units

OR

Orange flag: (5, 22.5)

39. i. Coordinates of the Treasure: (7, 14)
ii. Coordinates of the Police officer: (10, 7)
iii. Shortest distance between the Archaeologist and the Treasure:
Archaeologist is at (2, 2), Treasure is at (7, 14).

$$d = \sqrt{(7-2)^2 + (14-2)^2} = \sqrt{5^2 + 12^2} = \sqrt{25+144} = \sqrt{169} = 13$$

The shortest distance is 13 km.OR

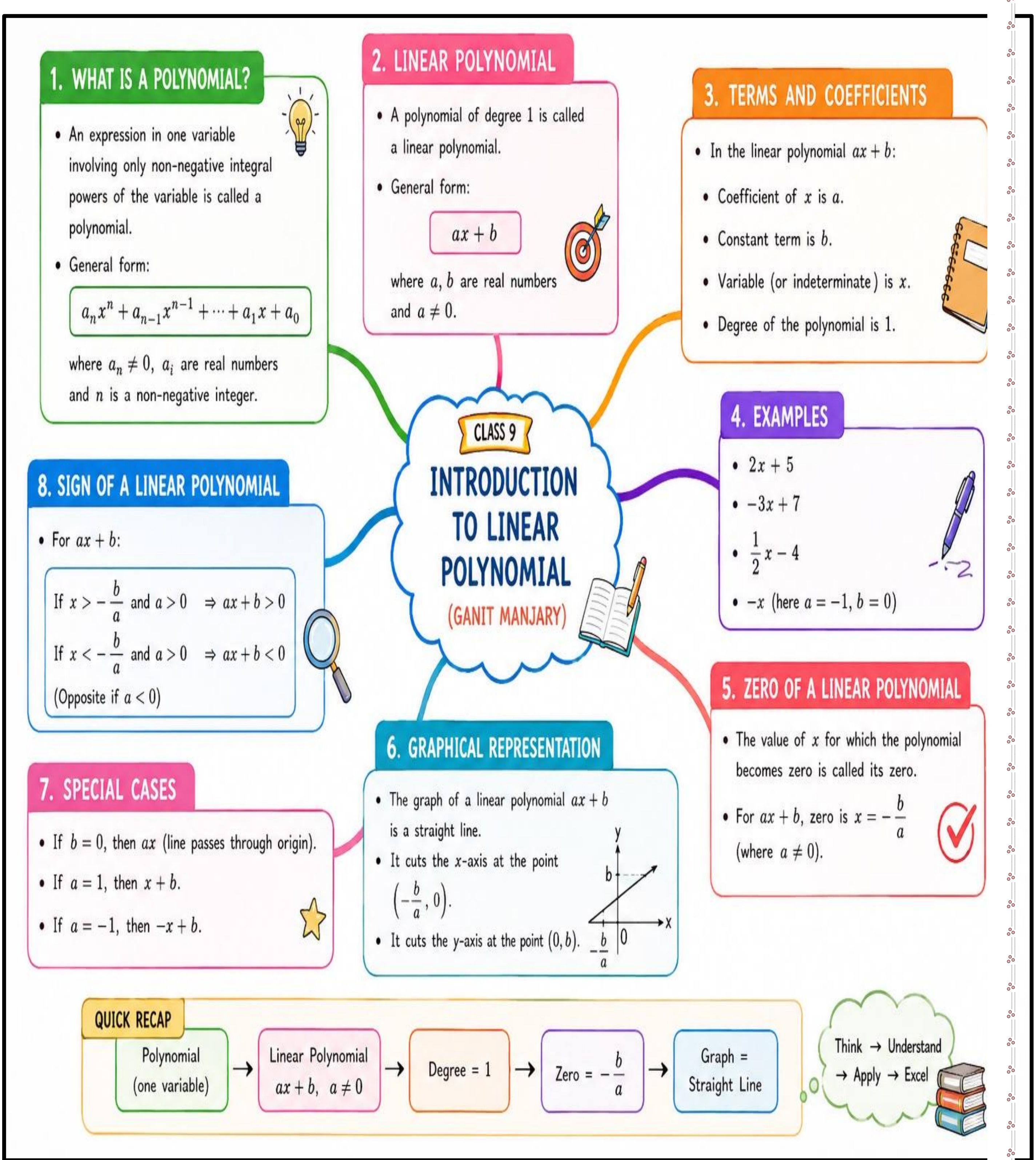
Distance between the Police officer and the Bandit:

The distance between the police officer and the bandit is √26 km.

CHAPTER2

INTRODUCTION TO LINEAR POLYNOMIALS

CONCEPT MAP



1. LEARNING OUTCOMES

1. Understand algebraic expressions, polynomials and their terms.
2. Identify the coefficient, the constant term and the degree of a polynomial.
3. Classify polynomials by degree (constant, linear, quadratic, cubic) and by number of terms (monomial, binomial, trinomial).
4. Understand the general form of a linear polynomial in one variable: $p(x) = ax + b$, where $a \neq 0$.
5. Find and verify the zero of a linear polynomial.
6. Represent a linear polynomial graphically as a straight line and interpret its x-intercept.

2. KEY CONCEPTS

2.1 Algebraic Expression

- A combination of constants and variables connected by the operations $+$, $-$, $\times$ and $\div$ is called an algebraic expression.
- Examples: $3x + 5$, $2x^2 - 7x + 1$, $4/x + y$.

2.2 Polynomial

- An algebraic expression in which the power of the variable in every term is a non-negative integer is called a polynomial.
- General form: $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, where n is a non-negative integer.

Note: *Expressions such as $\sqrt{x}$, $1/x$ or x^{-2} are not polynomials, since their powers are not non-negative integers.*

2.3 Terms and Coefficients

- The parts of a polynomial separated by + or − signs are called its terms.
- The numerical factor of a term is called its coefficient.
- A term that has no variable is called the constant term.

2.4 Degree of a Polynomial

- The highest power of the variable having a non-zero coefficient is called the degree of the polynomial.

Degree	Name	Example
0	Constant polynomial	$5, -3, 7$
1	Linear polynomial	$2x + 3, x - 5$
2	Quadratic polynomial	$x^2 - 4x + 4$
3	Cubic polynomial	$x^3 + 2x^2 - x$

2.5 Classification by Number of Terms

- Monomial – a polynomial with exactly one term (e.g. $5x$).
- Binomial – a polynomial with exactly two terms (e.g. $3x + 7$).
- Trinomial – a polynomial with exactly three terms (e.g. $x^2 + 2x + 1$).

3. LINEAR POLYNOMIAL

3.1 Definition and General Form

- A polynomial of degree 1 is called a linear polynomial.
- General form: $p(x) = ax + b$, where a and b are real numbers and $a \neq 0$.
- Examples: $2x + 3$, $x - 5$, $-3x$, $(x/2) + 1$.

Note: *If $a = 0$, the expression reduces to the constant b, which is not linear.*

3.2 Zero of a Linear Polynomial

- A zero of $p(x)$ is a value of x for which $p(x) = 0$.
- For $p(x) = ax + b$: $ax + b = 0 \Rightarrow x = -b/a$.
- A linear polynomial has exactly one zero, since the equation $ax + b = 0$ $(a \neq 0)$ has a unique solution.

3.3 Value of a Polynomial

- The value of $p(x)$ at $x = k$, written $p(k)$, is found by substituting k in place of x.
- Example: if $p(x) = 4x + 1$, then $p(2) = 4(2) + 1 = 9$.

3.4 Graph of a Linear Polynomial

- The graph of $y = ax + b$ ($a \neq 0$) is always a straight line.
- Steps: find at least two ordered pairs (x, y) by substitution, plot them on the Cartesian plane, and join them with a straight line.
- The line meets the x-axis at the point $(-b/a, 0)$; this x-coordinate is exactly the zero of the polynomial.
- **Section A: Multiple Choice Questions**

1. The degree of the polynomial $5y^3 + y^2 + 2y - 1$ is

(a) 1 (b) 2 (c) 3 (d) 0

Solution: The highest power of the variable y is 3. Hence the degree is 3.
Answer: (c)

2. The coefficient of x^2 in the polynomial $x^4 - 3x^3 + 6x^2 - 2x + 7$ is

(a) -3 (b) 6 (c) -2 (d) 7

Solution: The term containing x^2 is $6x^2$. Therefore the coefficient is 6.
Answer: (b)

3. Which of the following is a linear polynomial?

(a) $x^2 + 5x + 1$ (b) $3z + 7$ (c) $5y^3 + y^2 - 8$ (d) $x^4 - 3x^3$

Solution: A linear polynomial has degree 1. Only $3z + 7$ has degree 1.
Answer: (b)

4. The constant term in the polynomial $9x^3 + 5x^2 - 8x - 10$ is

(a) 9 (b) 5 (c) -8 (d) -10

Solution: The term independent of the variable is -10.

Answer: (d)

5. The value of the linear polynomial $5x - 3$ when $x = 2$ is

(a) 7 (b) 13 (c) -1 (d) 10

Solution: $5(2) - 3 = 10 - 3 = 7$.

Answer: (a)

6. If the number of square tiles at stage n is given by $2n - 1$, then the number of tiles at stage 15 is

(a) 29 (b) 30 (c) 31 (d) 28

Solution: $2 \times 15 - 1 = 30 - 1 = 29$.

Answer: (a)

7. The expression $200 + 50m$ represents the total cost for m matches. This is an example of

(a) quadratic polynomial (b) linear polynomial (c) cubic polynomial (d) constant polynomial

Solution: The degree of $200 + 50m$ is 1. Hence it is a linear polynomial.

Answer: (b)

8. In the equation $y = 2x + 1$, the slope is

(a) 1 (b) 2 (c) -2 (d) 0

Solution: In the form $y = ax + b$, a is the slope. Here $a = 2$.

Answer: (b)

9. The y-intercept of the line $y = 3x - 2$ is

(a) 3 (b) -2 (c) 2 (d) 0

Solution: The line cuts the y-axis at $(0, b)$. Here $b = -2$.

Answer: (b)

10. Which of the following represents linear growth?

(a) $h(t) = 3 - 0.5t$ (b) $C(d) = 100 + 60d$ (c) $10x - x^2$ (d) $5y^3 + 2y$

Solution: Linear growth occurs when a quantity increases by a constant amount (positive slope). $C(d) = 100 + 60d$ has positive slope 60.
Answer: (b)

11. The perimeter of a square of side x is

(a) x^2 (b) 4x (c) 2x (d) x + 4

Solution: Perimeter = 4 × side = 4x.
Answer: (b)

12. If y = ax + b and the line passes through the origin, then

(a) a = 0 (b) b = 0 (c) a = b (d) a = 1

Solution: When b = 0, the equation becomes y = ax and the line passes through (0, 0).
Answer: (b)

13. The difference between consecutive terms of the sequence 1, 3, 5, 7, 9, … is

(a) 1 (b) 2 (c) 3 (d) 0

Solution: Common difference = 2. This is a linear pattern.
Answer: (b)

14. The value of the quadratic polynomial $7s^2 - 4s + 6$ at $s = 0$ is

(a) 7 (b) −4 (c) 6 (d) 0

Solution: $7(0)^2 - 4(0) + 6 = 6$.

Answer: (c)

15. Lines with equal slopes but different y-intercepts are

(a) perpendicular (b) intersecting at origin (c) parallel (d) coincident

Solution: Same slope a and different b ⇒ parallel lines.

Answer: (c)

16. Which of the following is a constant polynomial?

(a) $4z - 3$ (b) -9 (c) $x^2 + 1$ (d) $2y - 5$

Solution: Constant polynomials have degree 0. -9 can be written as $-9x^0$.

Answer: (b)

Assertion-Reasoning Questions :

(a) Both (A) and (R) are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

17. Assertion (A): The polynomial $2x + 3$ is a linear polynomial.

Reason (R): The highest power of the variable in a linear polynomial is 1.

Solution: Both A and R are true and R correctly explains A.

Answer: (a)

18. Assertion (A): The graph of $y = -3x$ has a negative slope.

Reason (R): Linear decay is represented by a straight line with positive slope.

Solution: A is true (slope $= -3 < 0$). R is false because linear decay has negative slope.

Answer: (c)

19. Assertion (A): In the expression $y = 20x + 150$, the number 20 represents the fixed monthly fee.

Reason (R): In $y = ax + b$, a is the rate of change (slope) and b is the fixed charge (y-intercept).

Solution: A is false (20 is the cost per GB; 150 is the fixed fee). R is true.

Answer: (d)

20. Assertion (A): The sequence formed by the number of tiles 1, 3, 5, 7, … is a linear pattern.

Reason (R): The difference between consecutive terms is constant.

Solution: Both A and R are true and R correctly explains A.
Answer: (a)

Section B: Very Short Answer Type Questions

21. Find the degree and the constant term of the polynomial $4z^3 + 5z^2 - 11$.

Solution: Degree = highest power of z = 3. Constant term = -11.

Answer: Degree = 3, Constant term = -11

22. Write any one polynomial each of degree 1, 2 and 3.

Solution: Degree 1: $3x + 5$; Degree 2: $x^2 - 4x + 7$; Degree 3: $2y^3 + y - 1$. (Any correct examples are accepted.)

Answer: As above (or equivalent)

23. Evaluate the linear polynomial $5x - 3$ at $x = -1$.

Solution: $5(-1) - 3 = -5 - 3 = -8$.

Answer: -8

24. Express the total fare for travelling n km ($n \geq 2$) if the auto-rickshaw charges ₹25 for the first 2 km and ₹15 for every additional km, as a linear expression in n.

Solution: Fare = $25 + 15(n - 2) = 15n - 5$.

Answer: $15n - 5$

25. State the slope and y-intercept of the line $y = -2x + 5$.

Solution: In $y = ax + b$, slope $a = -2$ and y-intercept $b = 5$.

Answer: Slope = -2, y-intercept = 5

Section C: Short Answer Type Questions

26. Find the values of the linear polynomial $5x - 3$ for $x = 0$, $x = -1$ and $x = 2$.

Solution: for $x = 0$

$5(0) - 3 = -3$

For $x = -1$

$5(-1) - 3 = -8$

For $x = 2$

$5(2) - 3 = 7.$

Answer: $-3, -8, 7$

27. Draw the graph of the linear polynomial $p(x) = x - 2$ and find its zero from the graph.

Prepare a table of values:

x	0	2
$y = x - 2$	-2	0

Solution: Plotting the points $(0, -2)$ and $(2, 0)$ and joining them gives a straight line. The line crosses the x-axis at $(2, 0)$, so the zero of $p(x)$ read from the graph is $x = 2$, which matches $x - 2 = 0 \Rightarrow x = 2$.

28. Classify the following into monomial, binomial or trinomial, and state the degree of each: (i) $5x$ (ii) $3x + 7$ (iii) $x^2 + 2x + 1$ (iv) 9

Polynomial	**Type**	**Degree**
$5x$	Monomial	1
$3x + 7$	Binomial	1
$x^2 + 2x + 1$	Trinomial	2
9	Monomial	0

29. The present age of Salil's mother is three times Salil's present age. After 5 years, their ages will add up to 70 years. Find their present ages.

Solution: Let Salil's present age = x years.

Mother's age = $3x$ years.

After 5 years:

Salil's age will be = $x + 5$

Mother's age will be = $3x+5$

A.t.q $(x + 5) + (3x + 5) = 70$

$4x + 10 = 70$

$4x = 60$

$x = 15$.

Therefore Salil is 15 years and mother is 45 years old.

Answer: Salil: 15 years, Mother: 45 year

30. A student has ₹500 in her savings account. She receives ₹150 every month as pocket money. Write a linear expression for the amount she will have in the nth month and find the amount after 6 months.

Solution: Amount in nth month = $500 + 150n$.

After 6 months: $500 + 150 \times 6 = 500 + 900 =$ ₹1400.

Answer: Expression: $500 + 150n$;

Amount after 6 months = ₹1400

31. The height of water in a tank is given by $h(t) = 3 - 0.5t$. Find the height after 4 months and state whether it represents linear growth or linear decay.

Solution: After 4 months

$h(4) = 3 - 0.5 \times 4$

$= 3 - 2$

$= 1$ m.

It represents linear decay because the quantity decreases by a constant amount (0.5 m) each month.

Answer: Height = 1 m; Linear decay

Section D: Long Answer Type Questions

32. (i) Write a polynomial of degree 3 in x whose coefficient of x^2 is -7.

(ii) Find the value of $5x^2 - 3x + 7$ when $x = 1$.

iii) Find the coefficient of z in $4z^3 + 5z^2 - 11$.

Solution: (i) One possible polynomial: $x^3 - 7x^2 + 2x + 5$ (any correct example accepted). (ii) $5(1)^2 - 3(1) + 7 = 5 - 3 + 7 = 9$. (iii) There is no z^1 term, so the coefficient of z is 0.

Answer: (i) $x^3 - 7x^2 + \cdots$ (example); (ii) 9; (iii) 0

33. A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. Find the numbers.

Solution: Let the smaller number be x.

Then the larger number is 5x.

After adding 21:

New numbers are x + 21 and 5x + 21.

A .t.q. $5x + 21 = 2(x + 21)$

$5x + 21 = 2x + 42$

$3x = 21$

$x = 7.$

Therefore the numbers are 7 and 35.

Answer: 7 and 35

34. Draw the graph of the equation $y = 2x + 1$ by taking at least three points. Identify its slope and y-intercept. Also write the coordinates of the point where it cuts the y-axis.

Solution: Substituting x = 0 in the given equation we get y = 1

Substituting x = 1 in the given equation we get y = 3

Substituting x = 2 in the given equation we get y = 5

Substituting x = -1 in the given equation we get y = -1

x	0	1	2	-1
y	1	3	5	-1

Plot these points on the coordinate plane and join them with a straight line extending in both directions.

Slope = 2, y-intercept = 1. The line cuts the y-axis at (0, 1).

Answer: Slope = 2; y-intercept = 1; Cuts y-axis at (0, 1)

35. The graph of a linear polynomial p(x) passes through the points (1, 5) and (3, 11).

(i) Find the polynomial p(x).

(ii) Find the coordinates where the graph of p(x) cuts the axes.

Solution: Let $p(x) = ax + b$.

Atq $5 = a(1) + b = a + b$ ……….(1)

and $11 = a(3) + b = 3a + b$ ……….(2).

Subtracting: eq (1) from (2)

We get $6 = 2a$

$a = 3$.

Substituting value of a in eq. (1)

$5 = 3 + b$

$b = 2$.

Thus $p(x) = 3x + 2$.

intersects y-axis at $x = 0 \Rightarrow y = 2 \rightarrow (0, 2)$

Intersects x-axis at $y = 0 \Rightarrow 3x + 2 = 0 \Rightarrow x = -2/3 \rightarrow (-2/3, 0)$.

Answer: $p(x) = 3x + 2$; Cuts axes at $(0, 2)$ and $(-2/3, 0)$

Section E: Case-Based Questions

Case 1: Pocket Money

36. Bela has ₹100 for pocket money. She spends ₹5 every day. The amount on the nth day is given by $100 - 5n$.

(i) What amount will be left on the 15th day? (1 mark)

(ii) After how many days will the entire amount be spent? (1 mark)

(iii) Does this situation represent linear growth or linear decay? Justify by finding the slope of the corresponding line. (2 marks)

Solution: (i) $100 - 5 \times 15 = 100 - 75 = 25$.

(ii) $100 - 5n = 0$

$5n = 100$

$n = 20$ days.

(iii) It represents linear decay (amount decreases).

The slope of $y = 100 - 5n$ is -5 (negative), which confirms linear decay.

Answer: (i) ₹25; (ii) 20 days; (iii) Linear decay; slope = −5

Case 2: Temperature Conversion

37. The relation between temperature in Celsius (°C) and Fahrenheit (°F) is given by °C = a °F + b. Ice melts at 0 °C = 32 °F and water boils at 100 °C = 212 °F.

(i) Find the value of a. (1 mark)

(ii) Find the value of b. (1 mark)

(iii) Write the complete linear relationship and find the Celsius temperature when Fahrenheit is 68 °F. (2 marks)

Solution: (i) Using the points: When °F = 32, °C = 0

$$0 = 32a + b \text{ ....(1)}$$

and when °F = 212, °C = 100.

$$100 = 212a + b \text{ ....(2).}$$

Subtracting eq (1) from (2)

$$100 = 180a$$

$$\Rightarrow a = 5/9.$$

(ii) From (1): $b = -32 \times (5/9) = -160/9$.

(iii) Thus $°C = (5/9)(°F - 32)$.

When °F = 68:

$$°C = (5/9)(68 - 32)$$

$$= (5/9) \times 36$$

$$= 20.$$

Answer: (i) $a = 5/9$; (ii) $b = -160/9$; (iii) $°C = (5/9)(°F - 32)$; 20 °C

Case 3: Draining Water Tank

38. A water tank contains 500 litres of water. Water is drained at a constant rate of 25 litres per minute. The volume V (in litres) remaining after t minutes is given by $V(t) = -25t + 500$.

Questions:

(i) Identify the coefficient of t and the constant term in V(t).

(ii) Find the volume of water remaining after 8 minutes.

(iii) Find the zero of V(t) and explain what it represents physically.

Solutions:

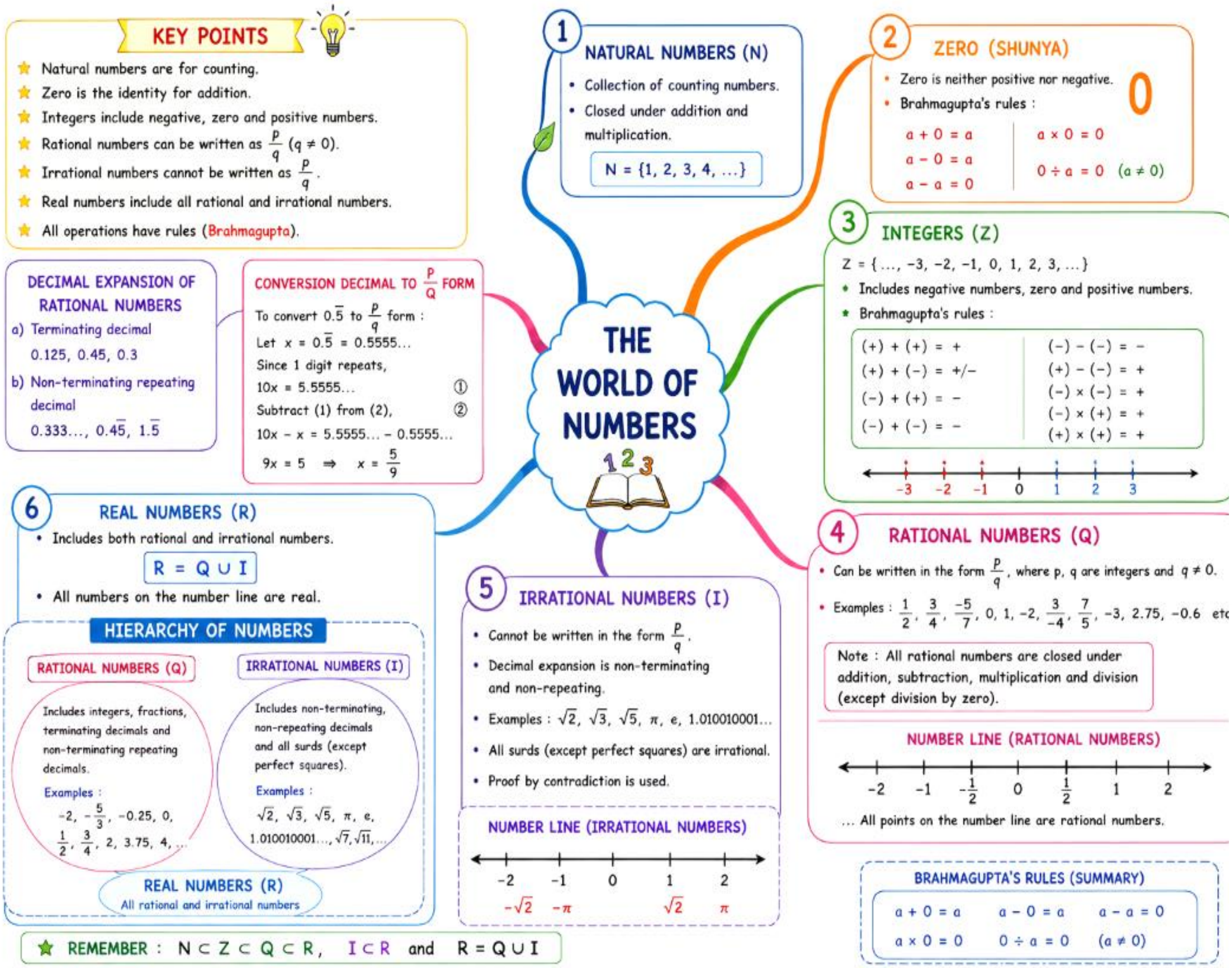
(i) Coefficient of $t = -25$; constant term = 500.

(ii) $V(8) = -25(8) + 500 = -200 + 500 = 300$ litres.

(iii) $-25t + 500 = 0 \Rightarrow t = 20$. This represents the time (20 minutes) at which the tank becomes completely empty.

CHAPTER 3 THE WORLD OF NUMBERS

MIND MAP



1. LEARNING OUTCOMES

1. Understand natural, whole, integer, rational, irrational and real numbers.
2. Learn the importance of zero (Shunya) and the contribution of Brahmagupta.
3. Perform operations on integers and rational numbers and represent them on the number line.
4. Understand the density property of rational and irrational numbers.
5. Differentiate between terminating and recurring decimals and understand real numbers as the combination of rational and irrational numbers

2. KEY CONCEPTS

2.1 Natural Numbers (N)

- Natural numbers (counting numbers) are the counting numbers: N = {1, 2, 3, 4, …}.
- Long ago, human needs numbers for counting animals, objects, grains and trade materials .
- Early counting methods included pebbles, sticks, tally marks on bones and fingers joints.
- Natural numbers are closed under addition and multiplication.

2.2 Zero (Shunya) and Brahmagupta

- Zero was one of the greatest developments in mathematics. Earlier, zero represented emptiness; Indian mathematicians gave it a mathematical value.
- Brahmagupta stated important rules for calculations involving zero:
- **Brahmagupta's Rules-** i) $a + 0 = a$ iii) $a \times 0 = 0$
  ii) $a - 0 = a$ iv) $a - a = 0$
- **Note**- Zero is neither positive nor negative. It is a whole number and an integer.

3. Integers (Z)

- Integers include negative numbers, zero and positive numbers.

  Z = {..., -3, -2, -1, 0, 1, 2, 3, ...}

3.1 Brahmagupta's interpretation:

- Positive numbers → fortunes / assets
- Negative numbers → debt / liabilities
- **Rules for signs:**

  i) (+) + (+) = + ii) (-) + (-) = - iii) (+) × (+) = + iv) (-) × (+) = - v) (-) × (-) = +

  Note- Integers are closed under addition, subtraction and multiplication

4. Rational Numbers (Q)

- A rational number is any number that can be written in the form p/q, where $q \neq 0$ and p and q are integers.

  Examples: 1/2, 3/4, -5/7, 0, 1, -2, 3/-4.

Note- Rational numbers are closed under addition, subtraction, multiplication and division, except division by zero.

4.1 Absolute Value

The absolute value of a rational number is always non-negative.

$|x| \geq 0$; $|-5| = 5$; $|6| = 6$.

4.2 Density of Rational Numbers

There are infinitely many rational numbers between any two rational numbers.

- A rational number between a and b is $\frac{a+b}{2}$, provided $a < b$.

4.3 Decimal Expansion of Rational Numbers

- Rational numbers give either terminating decimals or non-terminating recurring decimals.

Examples: $1/8 = 0.125$; $1/3 = 0.333...$; $5/11 = 0.454545...$

Note- If the denominator of a rational number in lowest form has only the prime factors 2 and/or 5, its decimal expansion is terminating.

Example: $\frac{3}{40} = \frac{3}{2^3 \times 5} = 0.75$.

4.4 Cyclic Numbers

- Some fractions produce repeating blocks of digits. For example, $1/7 = 0.142857142857...$. The recurring block is 142857.

$2/7 = 2 \times 0.\overline{142857}$

$3/7 = 3 \times 0.\overline{142857}$

4.5 Conversion of Recurring Decimal to p/q Form

Example: Convert 0.5555... into p/q form.

Let $x = 0.5555...$
$10x = 5.5555...$
$10x - x = 5.5555... - 0.5555... = 5$
$9x = 5$
$x = 5/9$

5. Irrational Numbers (I)

- Irrational numbers are numbers which cannot be written in the form p/q, where p and q are integers and $q \neq 0$.

Examples: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π, 1.1010010001...

Note- Their decimal expansion is non-terminating and non-repeating.

Examples- 3.1415926……., 1.4142135…….., 2.2020020002……… etc

Note- The square root of a natural number is irrational if the number is not a perfect square.

Examples: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ and $\sqrt{11}$ are irrational.

5.1 Proof of Irrationality

- Irrationality is commonly proved using the method of contradiction.

5.2 Construction of $\sqrt{n}$ on the Number Line

- $\sqrt{n}$ can be represented geometrically on the number line using right-triangle construction and Pythagoras' theorem.

6. Real Numbers (R)

- Real numbers include both rational and irrational numbers.
- Real Numbers = (Rational Numbers) ∪ (Irrational Numbers).

Note- All numbers represented on the number line are real numbers.

Number System Hierarchy

- Natural Numbers ⊂ Whole Numbers ⊂ Integers ⊂ Rational Numbers ⊂ Real Numbers

SECTION A – MCQs

1. Which of the following situations demonstrates that natural numbers are not closed under subtraction?

a) $8 - 3 = 5$ b) $15 - 10 = 5$ c) $4 - 7 = -3$ d) $15 - 4 = 11$

Solution: Option (c) is correct.

Explanation: The result −3 is not a natural number.

2. If a number x satisfies $x + 0 = x$, then which property is illustrated?

a) Rationality b) Identity c) Irrationality d) Closure

Solution: Option (b) is correct.

Explanation: Zero is the additive identity.

3. A lift moves from −4 to 3 and then to −2. What is the net movement of the lift?

a) 2 b) −1 c) 5 d) 7

Solution: Option (a) 2 is correct.

Explanation: Net movement = final position − initial position = $-2 - (-4) = 2$.

4. Which of the following is a rational number but not an integer?

a) −3 b) 0 c) 7 d) 5/8

Solution: Option (d) 5/8 is correct.

Explanation: 5/8 is in the form p/q, but it is not an integer.

5. The temperature in the high-altitude desert of Ladakh is recorded as 4°C at noon. By midnight, it drops by 15°C. What is the midnight temperature?

a) −11°C b) 11°C c) 19°C d) −19°C

Solution: Option (a) −11°C is correct.

Explanation: Midnight temperature = $4 - 15 = -11$°C.

6. Which statement is incorrect?

a) Debt × Fortune = Debt
b) Debt × Debt = Fortune
c) Debt ÷ Fortune = Debt
d) Fortune ÷ Debt = Fortune

Solution: Option (d) is marked as the incorrect statement in the provided material.

7. Which of the following is a rational number?

a) $\sqrt{3}$ b) π c) $\sqrt{5}$ d) $\sqrt{9}$

Solution: Option (d) $\sqrt{9}$ is correct.

Explanation: $\sqrt{9} = 3$, which is rational.

8. Which statement is correct?

a) All square roots are irrational.
b) Irrational numbers are finite decimals.
c) Only non-perfect square roots are irrational.
d) Irrational numbers are integers.

Solution: Option (c) is correct.

Explanation: $\sqrt{n}$ is irrational when n is not a perfect square (for positive integer n).

9. The solution of $-7/9 - (-2/3)$ is:

a) $-1/9$ b) $1/9$ c) $6/9$ d) $-5/9$

Solution: Option (a) $-1/9$ is correct.

Explanation: $-7/9 + 2/3 = -7/9 + 6/9 = -1/9$.

10. A tailor has 15 3/4 m of fine silk. If making one kurta requires 2 1/4 m of silk, exactly how many kurtas can be made?

a) 5 b) 7 c) 9 d) 11

Solution: Option (b) 7 is correct.

Explanation: $15\ 3/4 \div 2\ 1/4 = 63/4 \div 9/4 = 63/9 = 7$.

11. Which of the following is an irrational number?

a) 0.35 b) 2/5 c) 3.1415926… d) $\sqrt{49}$

Solution: Option (c) is correct.

Explanation: 3.1415926… is a non-terminating, non-recurring decimal.

12. Convert $2.0\overline{5}$ (i.e., 2.05555…) in the form of p/q.

a) 37/18 b) 41/20 c) 41/10 d) 37/90

Solution: Option (a) 37/18 is correct.

Explanation: Let x = 2.05555…; 10x = 20.5555… and 100x = 205.5555…. Subtracting gives 90x = 185, so x = 185/90 = 37/18.

13. Which rational number has a non-terminating decimal expansion?

a) 18/125 b) 7/40 c) 7/24 d) $45/(2^3 \times 5^2)$

Solution: Option (c) 7/24 is correct.

Explanation: $24 = 2^3 \times 3$, so its denominator contains a prime factor other than 2 and 5.

14. Which of the following numbers lies between 2/5 and 3/5?

a) 4/5 b) ½ c) 4/10 d) 6/10

Solution: Option (b) 1/2 is correct.

Explanation: 1/2 = 0.5, which lies between 2/5 = 0.4 and 3/5 = 0.6.

15. Which statement is incorrect / not always true?

a) Rational + Irrational = Irrational
b) Rational − Irrational = Irrational
c) Rational × Irrational = Irrational
d) Irrational × Irrational = Irrational

Solution: Option (d) is correct.

Explanation: For example, $\sqrt{2} \times 2\sqrt{2} = 4$, which is rational.

16. A rational number in its lowest form has denominator $2^3 \times 5$. How many decimal places will its decimal expansion have?

a) 2 b) 3 c) 4 d) 5

Solution: Option (b) 3 is correct.

Explanation: $2^3 \times 5 = 40$. Multiplying by 25 gives $1000 = 10^3$, so the decimal expansion terminates in 3 places.

Assertion and Reason Questions

Choose the correct option:
(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

17. Assertion: Every rational number has either a terminating or recurring decimal expansion.
Reason: A rational number can be expressed as p/q.

Solution: Option (a) is correct.

18. Assertion: The value of $2.\overline{36} + 0.\overline{23}$ when expressed in the simplest form is 257/99.
Reason: The square of an irrational number is always a rational number.

Solution: Option (c) is correct.

19. Assertion: The decimal representation of 3/18 is terminating.
Reason: If the denominator of a rational number is of the form $2^m \times 5^n$, where m and n are non-negative integers, then its decimal expansion is terminating.

Solution: Option (d) is correct.

20. Assertion: Every rational number is a real number.
Reason: Infinitely many irrational numbers lie between two irrational numbers.

Solution: Option (b) is correct.

SECTION B – Very Short Answer (VSA)

21. A trader records the following transactions: ₹200 earned, ₹350 lost and ₹158 earned again. Determine the final balance.

Solution: Final balance = ₹200 − ₹350 + ₹158 = ₹8. The final balance indicates a profit of ₹8.

22. A merchant in the port city of Lothal is exchanging bags of spices for copper ingots. He receives 15 ingots for every 2 bags of spices. If he brings 12 bags of spices to the market, how many copper ingots will he leave with?

Solution: 2 bags = 15 copper ingots.

Therefore, 1 bag = 15/2 ingots

and 12 bags = (15/2) × 12 = 90 copper ingots.

23. Simplify using the distributive property: $\frac{7}{9}\left[\frac{6}{7} - \frac{3}{4}\right]$

Solution: $\frac{7}{9}\left[\frac{6}{7} - \frac{3}{4}\right] = \frac{7}{9}\left[\frac{6}{7}\right] - \frac{7}{9}\left[\frac{3}{4}\right] = \frac{2}{3} - \frac{7}{12} = \frac{8}{12} - \frac{7}{12} = \frac{1}{12}$.

24. Find four rational numbers between 1/6 and 2/5.

Solution: LCM of 6 and 5 is 30.

Thus 1/6 = 5/30 and 2/5 = 12/30.

Four rational numbers are 7/30, 8/30, 9/30 and 10/30

(equivalently 7/30, 4/15, 3/10 and 1/3).

25. If $\frac{x}{3} + \frac{x}{5} = \frac{16}{15}$, find the rational number x.

Solution: LCM of 3 and 5 is 15.

Therefore, 5x/15 + 3x/15 = 16/15,

so 8x = 16 and x = 2.

SECTION C – Short Answer (SA)

26. Prove that √5 is an irrational number.

Solution: Assume √5 is rational.
Then √5 = p/q, where p and q are co-prime integers.
Squaring both sides
$5 = p^2/q^2$,
so $5q^2 = p^2$.
Thus 5 divides p^2, so 5 divides p.
Let p = 5k.
Then $5q^2 = 25k^2$,
giving $q^2 = 5k^2$.
Hence 5 divides q^2 and therefore 5 divides q.
Thus p and q have a common factor 5, contradicting that they are co-prime.
Therefore, our assumption is wrong
hence √5 is irrational.

27. Simplify: $(3\sqrt{5} - 5\sqrt{2})(4\sqrt{5} + 3\sqrt{2})$. State whether the result is rational or irrational, with justification.

Solution: $(3\sqrt{5} - 5\sqrt{2})(4\sqrt{5} + 3\sqrt{2}) = 3\sqrt{5}(4\sqrt{5} + 3\sqrt{2}) - 5\sqrt{2}(4\sqrt{5} + 3\sqrt{2})$

$\Rightarrow 3\sqrt{5}(4\sqrt{5}) + 3\sqrt{5}(3\sqrt{2})) - 5\sqrt{2}(4\sqrt{5}) - 5\sqrt{2}(3\sqrt{2})$

$\Rightarrow 12(5) + 9\sqrt{10} - 20\sqrt{10} - 30$

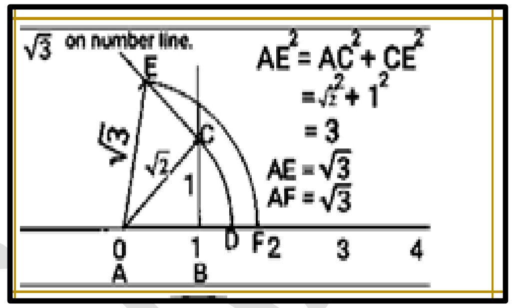
$\Rightarrow 60 - 30 - 11\sqrt{10}$

$\Rightarrow 30 - 11\sqrt{10}$

It is irrational number.

Justification: rational - irrational = irrational

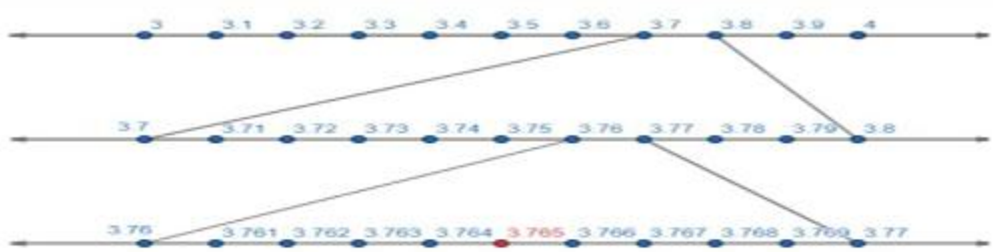
28. Represent √3 on the number line.



Solution: Take AB = 1 unit on the number line.
At B, draw a perpendicular BC = 1 unit.
Join OB. By Pythagoras, AC = $\sqrt{(1^2 + 1^2)} = \sqrt{2}$.
At C, draw a perpendicular CE = 1 unit
Join AE. By Pythagoras, AE = $\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{2+1} = \sqrt{3}$
With A as centre and AE as radius,
draw an arc cutting the number line at F. Then AF = $\sqrt{3}$.

29.Visualise 3.765 on the number line using successive magnification.

Solution: 3.765 can be visualised as in the following steps
First, we draw a number line and mark points on it
after that we will divide the number line between points 3 and 4.
And then we will divide the points between 3.7 and 3.8
as the number is between both of them.



30. Three rational numbers x, y, z satisfy x + y + z = 0 and xy + yz + zx = 0. Show that x, y, z must all be zero.

Solution: Given $x + y + z = 0$ and $xy + yz + zx = 0$

Squaring both sides $x + y + z = 0 \Rightarrow (x + y + z)^2 = (0)^2$

$\Rightarrow \quad x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0.$

Since $xy + yz + zx = 0$,

we get $x^2 + y^2 + z^2 = 0.$

A sum of squares of real (and hence rational) numbers is zero only when each square is zero.

Therefore $x = 0$, $y = 0$ and $z = 0$.

31. Convert $1.2\overline{35}$ (i.e., 1.2353535…) into the form of p/q.

Solution: Let $x = 1.2353535….$

Then $10x = 12.353535…$ equation (i)

and $1000x = 1235.353535….$ equation (ii)

Subtracting equation (i) from equation (ii)

$990x = 1223,$

Hence $x = 1223/990.$

SECTION D - Long Answer (LA)

32. Show that the rational number $\frac{a+b}{2}$ lies between the rational numbers *a and b*.

Solution: Let a and b be two rational numbers such that $a < b$.

We have to show that $a < \frac{a+b}{2} < b$

First part: since $a < b$.

Adding a to both sides

Gives $a + a < a + b,$

so $2a < a + b \Rightarrow a < \frac{a+b}{2}$ ……..(i)

Second part: since $a < b$.

Adding b to both sides

gives $a + b < b + b,$

so $(a + b) < 2b \Rightarrow \frac{a+b}{2} < b$ ……….(ii)

from (i) and (ii)

$a < \frac{a+b}{2} < b,$

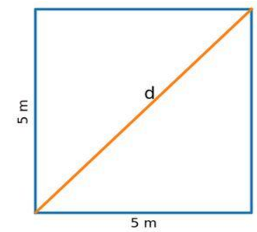
hence $\frac{a+b}{2}$ lies between *a and b.*

33. A square park has side 5 m. A diagonal path is constructed across it.

a) Find length of diagonal.

b) Identify that number is rational or irrational.

c) Represent the number on number line.



Solution: a)

The diagonal of a square forms a right angled triangle with two sides of the square

Using Pythagoras theorem:

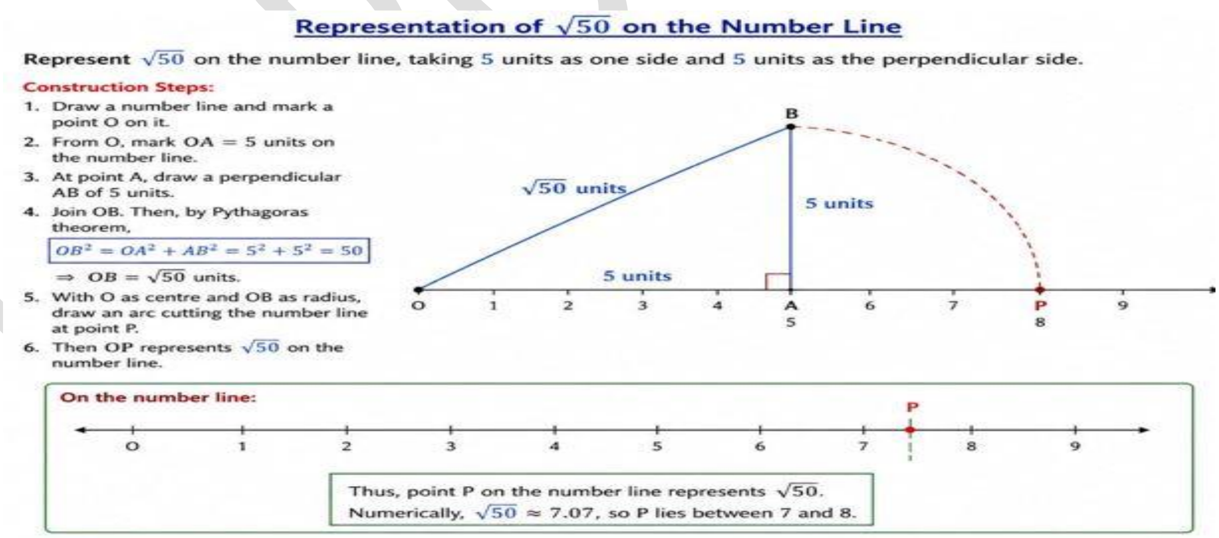
$(\text{Diagonal})^2 = \mathbf{5^2 + 5^2}$

$= \mathbf{25 + 25 = 50}$

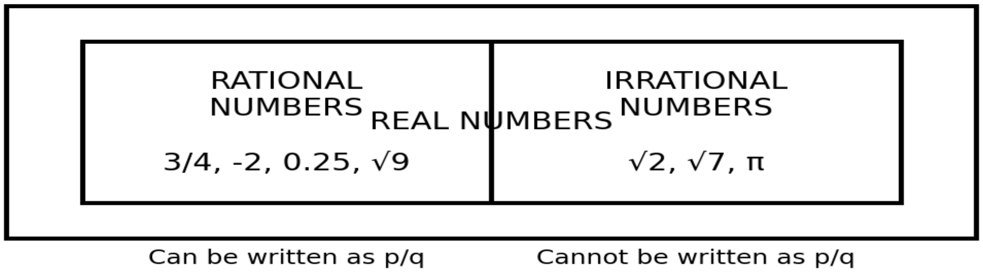
Diagonal $= \sqrt{\mathbf{50}} = \mathbf{5\sqrt{2}}$ **m**

b) since $\sqrt{2}$ *is irrational*, *so* $5\sqrt{2}$ *is also irrational*

c)



34. A mathematics teacher asks students to organise different numbers into two groups. Some numbers can be expressed in the form p/q, where p and q are integers and q ≠ 0, while others cannot. The students use the diagram to justify their classification.



(a) Which category does √9 belong to? Give its simplest value.

Solution: √9 = 3 so it is rational number. Its simplest value is 3**.**

(b) Which one of the following is irrational: 3/4, −2, 0.25 or √7?

Solution: √7 is irrational

(c) A student says that 0.3333… is irrational because its decimal expansion never ends. Do you agree? Justify your answer by identifying the type of decimal expansion.

Solution: No, 0.3333…… is a non terminating recurring decimal, so it is a rational.

0.33333….. = 1/3

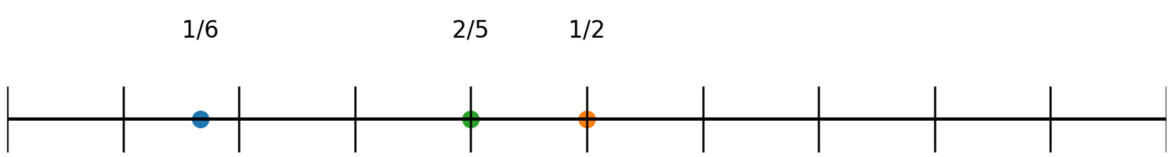
(d) Explain why √2 cannot be placed in the rational-number group.

Solution: √2 can not be placed in the rational number group because it can not be expressed in the form p/q , where p, q are integers and q ≠ 0.

Hence, √2 is irrational

35. Finding Rational Numbers Between Two Numbers

A teacher asks students to locate rational numbers between 1/6 and 2/5 on a number line. The students observe that there are many possible rational numbers between two distinct rational numbers.



A rational number can be placed between any two distinct rational numbers.

(a). Find the average of 1/6 and 2/5.

Solution : (1/6 + 2/5) ÷ 2 = (5/30 + 12/30) ÷ 2 = 17/60 ÷ 2 = 17/120.

(b). Is 1/2 between 1/6 and 2/5? Give a reason.

Solution : No. 1/2 = 0.5, while 2/5 = 0.4; hence 1/2 is greater than 2/5.

(c) Find any four rational numbers between 1/6 and 2/5.

Solution: Using denominator 30: 1/6 = 5/30 and 2/5 = 12/30. Four examples are 6/30, 7/30, 8/30 and 9/30.

(d) Explain why there are infinitely many rational numbers between two distinct rational numbers.

Solution : If a and b are two distinct rational numbers, their average (a+b)/2 is another rational number between them. This process can be repeated indefinitely, so infinitely many rational numbers lie between them.

SECTION E – Case Study Based Question

36. Case Study 1: Number Game

Two friends were given a project to tag the numbers given to them as R, IR which are short form for rational number, irrational number respectively. The numbers are: 1.707007000….., 1/5, √49, √2 , √3, 5.020020002….., ¾, √7, $\frac{\sqrt{54}}{\sqrt{6}}$ and $\frac{1}{2\sqrt{3}-\sqrt{11}}$,

Questions:

(i) Identify rational and irrational numbers from the given numbers. .

(ii) Find two irrational numbers between √2 and √3.

(iii)(a) Rationalise $\frac{1}{2\sqrt{3}-\sqrt{11}}$

OR

(iii) (b) Simplify: $(3-5\sqrt{2})(4+3\sqrt{2})$.

Solutions:

(i) Rational numbers: 1/5, √49 = 7 (since √49 is a perfect square), ¾ and $\frac{\sqrt{54}}{\sqrt{6}}$

Irrational numbers: 1.707007000… (non-terminating and non-repeating), √2, √3, 5.020020002… (non-terminating and non-repeating) and $\frac{1}{2\sqrt{3}-\sqrt{11}}$,

(ii) Two irrational numbers between √2 and √3 are 1.5050050005…….. and 1.6060060006………

(iii)(a) $\frac{1}{2\sqrt{3}-\sqrt{11}} = \frac{2\sqrt{3}+\sqrt{11}}{(2\sqrt{3}-\sqrt{11})(2\sqrt{3}+\sqrt{11})} = \frac{2\sqrt{3}+\sqrt{11}}{(2\sqrt{3})^2-(\sqrt{11})^2} = \frac{2\sqrt{3}+\sqrt{11}}{12-11} = 2\sqrt{3}+\sqrt{11}$

OR

(iii)(b) $(3-5\sqrt{2})(4+3\sqrt{2}) = 3\ (4)\ + 3\ (3\sqrt{2})) - 5\sqrt{2}\ (4)\ - 5\sqrt{2}\ (3\sqrt{2})$

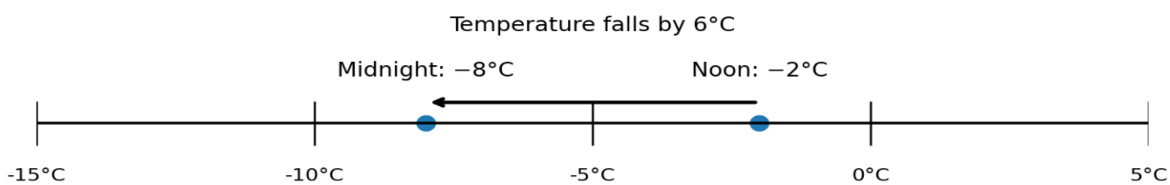
$\Rightarrow 12 + 9\sqrt{2} - 20\sqrt{2} - 30$

$\Rightarrow 12 - 30 - 11\sqrt{2}$

$\Rightarrow -8 - 11\sqrt{2}$

37. Case Study 2 – Temperature Change in a Cold Region

A weather station records the temperature of a cold region. At noon, the temperature is −2°C. By midnight, it falls by 6°C. The number line below represents the change in temperature.



(i). What is the temperature at midnight?

Solution : −2 − 6 = −8°C.

(ii). Which property of integers is illustrated when the temperature changes are represented using addition/subtraction?

Solution : The integers are closed under addition and subtraction.

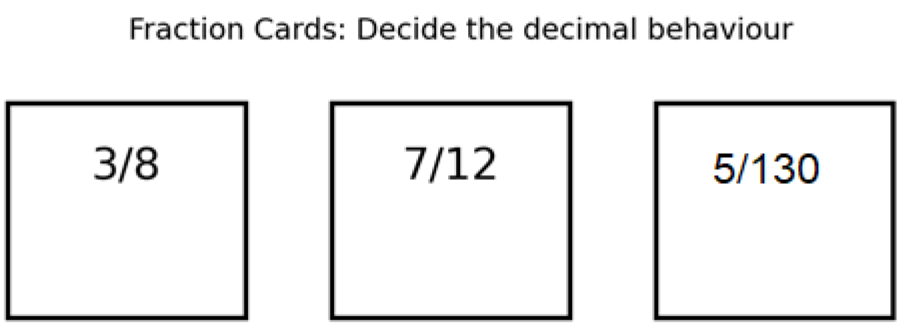
(iii). (a) If the temperature rises by 10°C from −8°C, what will be the new temperature?

Solution (a) : −8 + 10 = 2°C. **OR**

(iii) (b) A student says that natural numbers are closed under subtraction because 8 − 3 = 5. Is the statement always true? Justify with a suitable example.

Solution (b): No. Natural numbers are not closed under subtraction. For example, 4 − 7 = −3, which is not a natural number.

38. CASE STUDY 3 - **A shopkeeper has three fraction cards for a classroom numeracy game. Students must predict whether each fraction will have a terminating or a non-terminating recurring decimal expansion. They are reminded that the fraction should first be written in lowest form.**



(i) Which card represents a fraction with a terminating decimal expansion?

Solution: 3/8

(ii) State the prime factors of the denominator of 3/40.

Solution: $40 = 2 \times 2 \times 2 \times 5$

(iii) (a) A student claims that 7/12 will have a terminating decimal because 12 is a finite number. Check the claim using the prime-factor rule and justify your answer.

Solution: $12 = 2 \times 2 \times 3$

For a fraction in lowest form to have a terminating decimal, it denominator must have only 2 and 5 (2 or 5) as prime factors. Since 12 contains the prime factor 3, so it is non- terminating recurring

OR

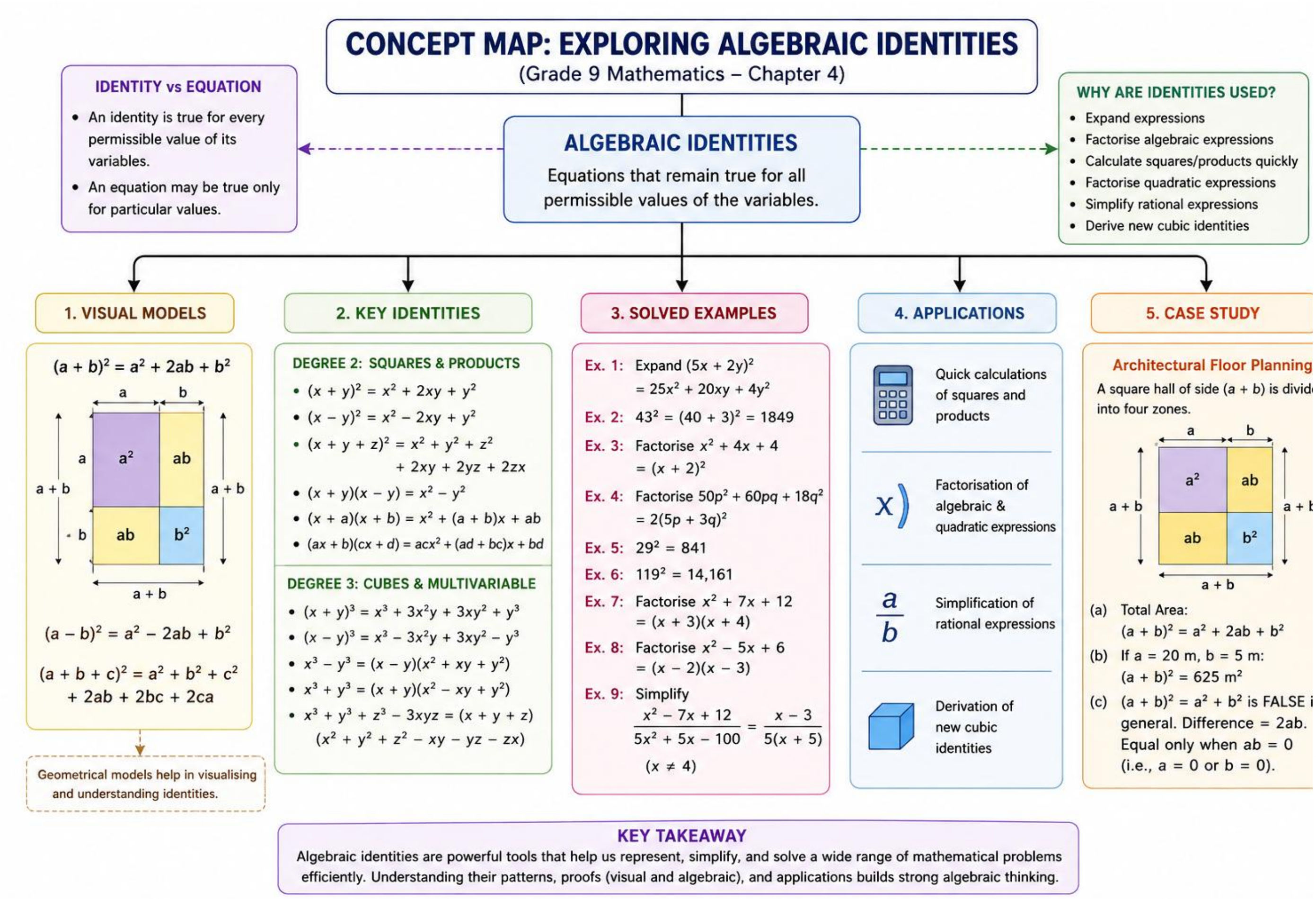
(iii) (b) Determine whether 5/130 has a terminating or non-terminating recurring decimal expansion and explain why.

Solution: simplest form of fraction 5/130 = 1/26

Factors of $26 = 2 \times 13$

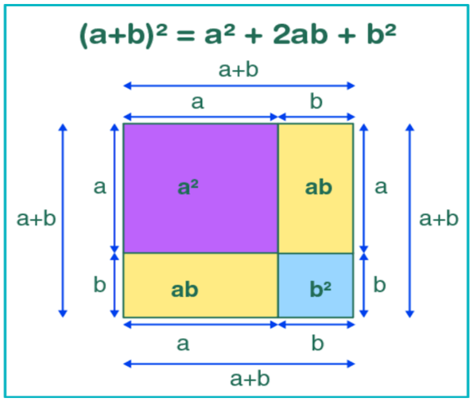
For a fraction in lowest form to have a terminating decimal, it denominator must have only 2 and 5 (2 or 5) as prime factors. Since 26 contains the prime factor 13, so it is non- terminating recurring decimal.

Chapter 4 – Exploring Algebraic Identities



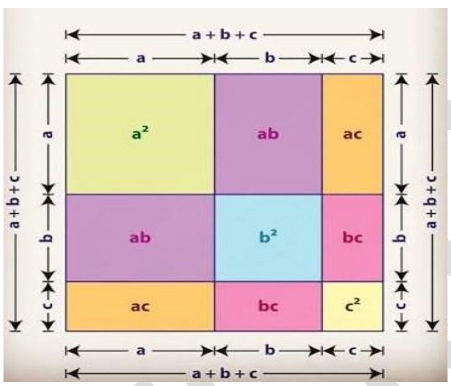
The chapter develops algebraic identities as equations that remain true for all values of the variables. It begins with patterns in consecutive square numbers and then visualises identities through geometrical models. The chapter uses identities to expand expressions, factorise algebraic expressions, calculate numerical squares/products quickly, factorise quadratic expressions, simplify rational expressions, and derive new cubic identities.

- An identity is true for every permissible value of its variables, whereas an equation may be true only for particular values.
- The identity $(a + b)^2 = a^2 + 2ab + b^2$ can be visualised by dividing a square of side $(a + b)$ into two squares and two rectangles.

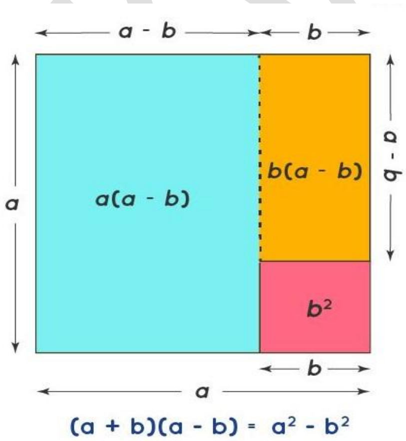


- Replacing b by −b gives $(a - b)^2 = a^2 - 2ab + b^2$.

- For three terms: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.



- Difference of squares: $a^2 - b^2 = (a + b)(a - b)$.



- General quadratic product: $(x + a)(x + b) = x^2 + (a + b)x + ab$.

- The identity $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - xz - yz)$ is used in applications

Key Identities – Quick Revision Sheet

DEGREE 2: SQUARES & PRODUCTS

1. Square Expansions

.$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

2. Product Expansions

- $(x + y)(x - y) = x^2 - y^2$
- $(x + a)(x + b) = x^2 + (a+b)x + ab$
- $(ax + b)(cx + d) = acx^2 + (ad+bc)x + bd$

DEGREE 3: CUBES & MULTIVARIABLE

1. Binomial Cubes

- $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
- $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

2. Sum and Difference of Cubes

- $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
- $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Multivariable Cubic Form

- $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Solved Examples

Example 1: Expand $(5x + 2y)^2$

Use $(a + b)^2 = a^2 + 2ab + b^2$

Here $a = 5x$ and $b = 2y$

$(5x + 2y)^2 = (5x)^2 + 2(5x)(2y) + (2y)^2$

$= 25x^2 + 20xy + 4y^2$

Example 2: Find 43^2 using an identity

Write $43 = 40 + 3$

$43^2 = (40 + 3)^2 = 40^2 + 2(40)(3) + 3^2$

$= 1600 + 240 + 9 = 1849$

Example 3: Factorise $x^2 + 4x + 4$

Compare with $a^2 + 2ab + b^2$

$x^2 + 4x + 4 = x^2 + 2(x)(2) + 2^2$

$= (x + 2)^2$

Example 4: Factorise $50p^2 + 60pq + 18q^2$

$50p^2 + 60pq + 18q^2 = 2(25p^2 + 30pq + 9q^2)$

$25p^2 = (5p)^2$, $9q^2 = (3q)^2$, and $30pq = 2(5p)(3q)$

$=2(5p + 3q)^2$

Example 5: Calculate 29^2 using $(a - b)^2$

Write $29 = 30 - 1$

$29^2 = 30^2 - 2(30)(1) + 1^2$

$= 900 - 60 + 1$

$= 841$

Example 6: Find 119^2 using $(a + b + c)^2$

Write $119 = 100 + 10 + 9$

Use identity: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$119^2 = 100^2 + 10^2 + 9^2 + 2(100)(10) + 2(10)(9) + 2(100)(9)$

$= 10{,}000 + 100 + 81 + 2{,}000 + 180 + 1{,}800$

$= 14{,}161$

Example 7: Factorise $x^2 + 7x + 12$

Find two numbers whose sum is 7 and product is 12

The numbers are 3 and 4

Split $7x$ as $3x + 4x$

$x^2 + 7x + 12 = x^2 + 3x + 4x + 12 = x(x + 3) + 4(x + 3)$

$= (x + 3)(x + 4)$

Example 8: Factorise $x^2 - 5x + 6$

Find two numbers whose sum is -5 and product is 6

They are -2 and -3

$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$

$= x(x - 2) - 3(x - 2)$

$= (x - 2)(x - 3)$

Example 9: Simplify $(x^2 - 7x + 12) / (5x^2 + 5x - 100)$

Factor numerator: $x^2 - 7x + 12 = (x - 3)(x - 4)$

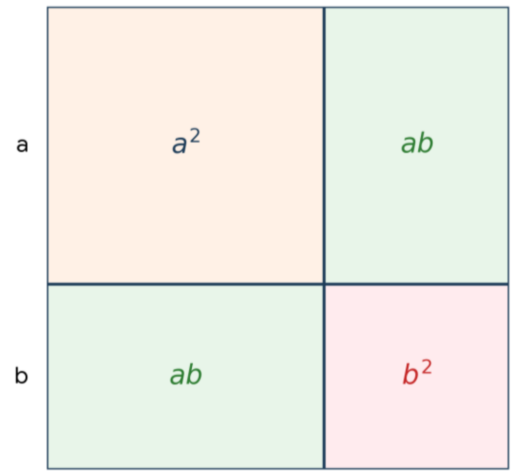
Factor denominator: $5x^2 + 5x - 100 = 5(x^2 + x - 20) = 5(x - 4)(x + 5)$

Cancel the common factor $(x - 4)$, provided $x \neq 4$

Answer : $(x - 3) / [5(x + 5)]$

Case Study 1: Architectural Floor Planning & Visualising Identities

An architect is designing a square community hall of total side length (a + b) metres. To make efficient use of space, the floor plan is partitioned into four functional zones: a large square auditorium of side 'a' metres, a small square storage room of side 'b' metres, and two identical rectangular exhibition corridors, each having dimensions 'a' metres by 'b' metres



a)Write an algebraic expression for the total area of the community hall in two different ways (as a single square and as the sum of its four partitioned zones).

b) If the dimensions are chosen as a = 20 metres and b = 5 metres, calculate the total area using the algebraic identity $(a + b)^2 = a^2 + 2ab + b^2$.

c) State whether $(a + b)^2 = a^2 + b^2$ is true. If a = 10 and b = 2, find by how much $(a + b)^2$ exceeds $(a^2 + b^2)$. Under what condition will $(a + b)^2 = a^2 + b^2$?

<u>**Solution:**</u>

(a) Total Area Expressions:

As a single square of side (a + b): Total Area = $(a + b)^2$ sq. metres.

Sum of four partitioned zones: Area = Area(Auditorium) + Area(Storage) + 2 × Area(Corridor) = $a^2 + b^2 + 2ab$ sq. metres.

Equating both gives the standard algebraic identity: $(a + b)^2 = a^2 + 2ab + b^2$

(b) Calculation for a = 20 m and b = 5 m:

$(a + b)^2 = (20 + 5)^2 = 25^2 = 625$ sq. metres.

Using expansion: $a^2 + 2ab + b^2 = (20)^2 + 2(20)(5) + (5)^2 = 400 + 200 + 25 = 625$ sq. metres.

(c) Comparison of $(a + b)^2$ and $(a^2 + b^2)$:

Statement: $(a + b)^2 = a^2 + b^2$ is FALSE in general because $(a + b)^2 - (a^2 + b^2) = 2ab$.

For $a = 10$ and $b = 2$:

$(a + b)^2 = (10 + 2)^2 = 144$

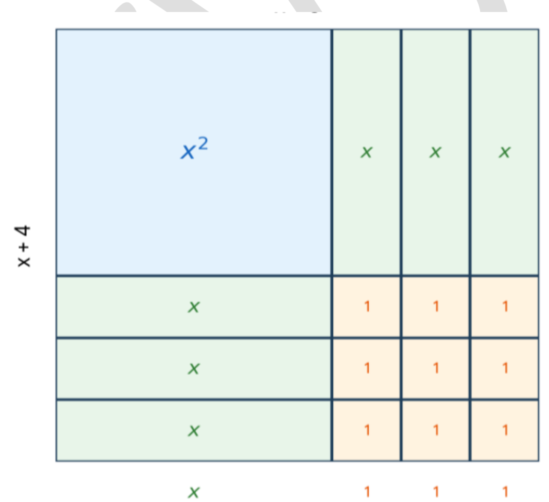
$a^2 + b^2 = 10^2 + 2^2 = 100 + 4 = 104$

Difference = 144 - 104 = 40 (which equals $2ab = 2 \times 10 \times 2 = 40$).

Condition for Equality: $(a + b)^2 = a^2 + b^2$ holds true IF AND ONLY if $2ab = 0$, which implies either $a = 0$ or $b = 0$.

Case Study 2: Tiling Layout & Polynomial Factorisation

A tile craftsman, Saira, is laying out decorative rectangular patterns using modular algebra tiles. She forms a rectangular surface using 1 large square x^2-tile, 7 rectangular x-tiles, and 12 small unit square tiles. To arrange them compactly into a solid seamless rectangle without overlaps, she places 3 x-tiles along one side and 4 x-tiles along the adjacent side, filling the corner with the 12 unit tiles in a 3 × 4 grid array.



(a) Express the total area of the tiles arranged by Saira as a quadratic polynomial in x, and write its factored form representing the dimensions (length and breadth) of the rectangle.

(b) Factorise the expression $x^2 + 11x + 30$ by splitting the middle term without using tiles, explaining the choice of factors.

(c) Simplify the rational expression $(x^2 - 7x + 12) / (5x^2 + 5x - 100)$, assuming the denominator is non-zero.

Solution

(a) Area & Factored Form:

Total Area = $x^2 + 3x + 4x + 12 = x^2 + 7x + 12$ sq. units.

Dimensions from diagram: Length = $(x + 4)$ units, Breadth = $(x + 3)$ units.

Factored Identity: $x^2 + 7x + 12 = (x + 3)(x + 4)$.

(b) Factorisation of $x^2 + 11x + 30$:

We compare with $x^2 + (a + b)x + ab$:

Sum $(a + b) = 11$, Product $(ab) = 30$.

Factors of 30 are (1, 30), (2, 15), (3, 10), (5, 6). The pair with sum 11 is 5 and 6.

Splitting 11x: $x^2 + 5x + 6x + 30 = x(x + 5) + 6(x + 5) = (x + 5)(x + 6)$.

(c) Simplification of Rational Expression:

Numerator: $x^2 - 7x + 12 = (x - 3)(x - 4)$ [since $-3 + (-4) = -7$, $(-3)(-4) = 12$].

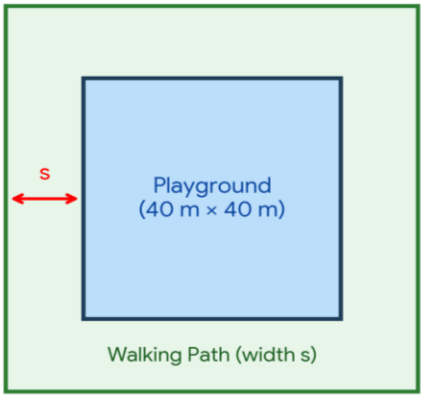
Denominator: $5x^2 + 5x - 100 = 5(x^2 + x - 20) = 5(x + 5)(x - 4)$ [since $5 + (-4) = 1$, $(5)(-4) = -20$].

Rational Expression = $[(x - 3)(x - 4)] / [5(x + 5)(x - 4)]$.

Cancelling common factor $(x - 4) \neq 0$ yields: $(x - 3) / [5(x + 5)]$ or $(x - 3) / (5x + 25)$.

Case Study 3: Village Playground & Border Pathway Design

A village gram panchayat constructs a central square playground of side length 40 metres. To facilitate morning walks for residents, a paved pathway of uniform width 's' metres is built all around the outside of the playground, forming a larger outer square region.



(a) Write the side length of the outer boundary square in terms of 's'. Derive a simplified algebraic expression for the area of the walking path.

(b) If the width of the path is s = 3 metres, compute the area of the path using the derived formula as well as the identity $a^2 - b^2 = (a + b)(a - b)$.

(c) The village also plans a rectangular swimming pool near the park whose area is given by $(2x^2 + 7x + 3)$ square hastas. If its width is $(2x + 1)$ hastas, find an expression for its length.

Solution:

(a) Expression for Path Area:

- Inner playground side = 40 m.
- Outer square side = 40 + s + s = (40 + 2s) m.
- Area of Path = Area(Outer Square) - Area(Inner Playground)

  $= (40 + 2s)^2 - 40^2$

  $= (1600 + 160s + 4s^2) - 1600 = 4s^2 + 160s$ sq. metres.

(b) Calculation for s = 3 m:

Outer side = 40 + 2(3) = 46 m.

Using Identity $a^2 - b^2 = (a + b)(a - b)$:

Area $= 46^2 - 40^2 = (46 + 40)(46 - 40) = (86)(6) = 516$ sq. metres.

Direct substitution into $4s^2 + 160s$:

Area $= 4(3)^2 + 160(3) = 4(9) + 480 = 36 + 480 = 516$ sq. metres. (Matches verified!)

(c) Pool Length Calculation:

Area = Length × Width => $2x^2 + 7x + 3 = \text{Length} \times (2x + 1)$.

Factorising $2x^2 + 7x + 3$:

Splitting $7x$ into $6x + x$ (since $6 \times 1 = 6 = 2 \times 3$ and $6 + 1 = 7$):

$2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$.

Comparing with Width × Length: Length $= (x + 3)$ hastas.

<u>MCQ'S</u>

<u>Q</u>1. What is the main difference between an algebraic equation and an algebraic identity?

(a) An equation is true for all real values of the variables, while an identity is true for only specific values.

(b) An identity is true for all values of the variables occurring in it, while an equation need not be true for
all values.

(c) An equation always involves two variables, while an identity involves only one variable.

(d) There is no mathematical difference between an equation and an identity.

Q2. If we take any three consecutive square numbers (e.g., 9, 16, 25), add the smallest and the largest squares, and subtract twice the middle square, what is the result?

(a) 0 (b) 1 (c) 2 (d) 4

Q3. What is the expansion of $(5x + 2y)^2$?

(a) $25x^2 + 4y^2$ (b) $25x^2 + 10xy + 4y^2$

(c) $25x^2 + 20xy + 4y^2$ (d) $10x^2 + 20xy + 4y^2$

Q4. Which of the following expressions represents the expansion of $(a + b + c)^2$?

(a) $a^2 + b^2 + c^2 + ab + bc + ca$

(b) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(c) $a^3 + b^3 + c^3 + 3abc$

(d) $a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$

Q5. What are the factors of $36x^2 + 12x + 1$?

(a) $(6x - 1)^2$ (b) $(6x + 1)(6x - 1)$

(c) $(6x + 1)^2$ (d) $(12x + 1)(3x + 1)$

Q6. Using the algebraic identity $(a - b)(a + b) = a^2 - b^2$, evaluate 104×96.

(a) 9984 (b) 9976 (c) 10016 (d) 9996

Q7. Which linear factors multiply to give the quadratic expression $x^2 + 7x + 12$?

(a) $(x + 2)(x + 5)$ (b) $(x + 3)(x + 4)$

(c) $(x - 3)(x - 4)$ (d) $(x + 1)(x + 12)$

Q8. What is the expansion of the cubic binomial $(2n - 5m)^3$?

(a) $8n^3 - 125m^3$ (b) $8n^3 - 60n^2m + 150nm^2 - 125m^3$

(c) $8n^3 + 60n^2m + 150nm^2 + 125m^3$ (d) $8n^3 - 30n^2m + 30nm^2 - 125m^3$

Q9. Simplify the rational algebraic expression: $(x^2 - 7x + 12) / (5x^2 + 5x - 100)$ (assuming denominator $\neq 0$).

(a) $(x - 3) / [5(x + 5)]$ (b) $(x - 4) / [5(x - 5)]$

(c) $(x + 3) / [5(x + 5)]$ (d) $(x - 3) / (x + 5)$

Q10. If $x + y + z = 10$, $x^2 + y^2 + z^2 = 38$, and $xyz = 25$, find the value of $x^3 + y^3 + z^3$.

(a) 120 (b) 145 (c) 310 (d) 455

<u>Answer key</u>:

Q1. (b)

Q2. (C) 2

Q3. (C) $25x^2 + 20xy + 4y^2$

Q4) b) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Q5. (C) $(6x + 1)^2$

Q6. (a) 9984

Q7. (b) $(x + 3)(x + 4)$

Q8.(b) $8n^3 - 60n^2m + 150nm^2 - 125m^3$

Q9. (a) $(x - 3) / [5(x + 5)]$

Q10.(b) 145

ASSERTION & REASONING QUESTIONS

Directions: In each of the following questions, a statement of Assertion (A) is followed by a statement of
Reason (R). Choose the correct option from the following:

(a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is NOT the correct explanation of Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

Question 1

Assertion (A): The expression (x + 5) 2 = x 2 + 10x + 25 is an algebraic identity.

Reason (R): An algebraic identity is an equality relation that remains true for all real values of the variables involved in it.

Question 2

Assertion (A): If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

Reason (R): The algebraic identity states that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

Question 3

Assertion (A): The value of 105×95 calculated using algebraic identities is 9975.

Reason (R): The identity used to solve 105 × 95 is $(a + b)^2 = a^2 + 2ab + b^2$.

Question 4

Assertion (A): The factors of $4x^2 - 12x + 9$ are $(2x - 3)^2$.
Reason (R): Any quadratic expression of the form $a^2 - 2ab + b^2$ can be factored into $(a - b)^2$.

Question 5

Assertion (A): The expansion of $(x - y)^3$ is $x^3 - y^3 - 3xy(x + y)$.
Reason (R): The standard identity for the cube of a binomial difference is $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

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ANSWER KEY

Q1. (a) Q2.(a) Q3.(c) Q4.(a) Q5(d)

Practice Questions

Question 1: Expand $(7x + 4y)^2$

Solution:

$\mathbf{(7x + 4y)^2} = (7x)^2 + 2(7x)(4y) + (4y)^2$

$= 49x^2 + 56xy + 16y^2$

Question 2: Find 105^2 using an identity

Solution:

$105 = 100 + 5$

$105^2 = (100 + 5)^2 = 10000 + 1000 + 25$

$=11025$

Question 3: Find 79^2 using $(a – b)^2$.

Solution:

$79 = 80 – 1$

$79^2 = (80 - 1)^2 = 80^2 - 2(80)(1) + 1^2$

$= 6400 - 160 + 1 = 6241$

Question 4: Factorise $9x^2 + 24xy + 16y^2$.

Solution:

Recognise it in the form $a^2 + 2ab + b^2$ as $(3x)^2 + 2(3x)(4y) + (4y)^2$ = $\mathbf{(3x + 4y)^2}$

Question 5: Factorise $16y^2 - 24y + 9$.

Solution:

Recognise it in the form $a^2 - 2ab + b^2$ as $(4y)^2 - 2(4y)(3) + 3^2$ **= $(4y - 3)^2$**

Question 6: Factorise $4s^2 + 20st + 25t^2$

Solution:

$4s^2 + 20st + 25t^2$ $= (2s)^2 + 2(2s)(5t) + (5t)^2$ **= $(2s + 5t)^2$**

Question 7: Factorise $36x^2 + 12x + 1$

Solution:

$36x^2 + 12x + 1$ $= (6x)^2 + 2(6x)(1) + 1^2$ **= $(6x + 1)^2$**

Question 8: Factorise $x^2 + 11x + 30$

Solution:

Find factors of 30 whose sum is 11: 5 and 6.

$x^2 + 11x + 30$ $= x^2 + 5x + 6x + 30$ **= $(x + 5)(x + 6)$**

Question 9: Factorise $x^2 - 5x + 6$

Solution:

Find factors of 6 whose sum is −5: −2 and −3.

Answer: $x^2 - 5x + 6 = (x - 2)(x - 3)$

Question 10: Expand $(p + 3q + 7r)^2$

Solution:

Use identity: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\mathbf{(p + 3q + 7r)^2} = p^2 + (3q)^2 + (7r)^2 + 2(p)(3q) + 2(3q)(7r) + 2(7r)(p)$

$= p^2 + 9q^2 + 49r^2 + 6pq + 42qr + 14pr$

Practice Questions – Mixed Revision

1. Expand $(3x - 2y + 4z)^2$.

2. Find 193^2 using $(a - b)^2$.

3. Factorise $49g^2 + 14gh + h^2$.

4. Factorise $r^2 - r - 42$.

5. Factorise $x^3 - 27$.

6. Factorise $x^3 + 8$.

7. Expand $(2a - 3b)^3$.

8. Find the side of a cube whose volume is $p^3 + 6p^2q + 12pq^2 + 8q^3$.

9. If $x + y + z = 10$, $xyz = 25$ and $x^2 + y^2 + z^2 = 38$, find $x^3 + y^3 + z^3$.

10. A square playground has side 40 m and a path of width s m is built around it. Express the area of the path.

Answer KEY

1. $9x^2 + 4y^2 + 16z^2 - 12xy - 16yz + 24xz$ (Note: corrected xz and yz signs from source key)

2. $193^2 = (200 - 7)^2 = 200^2 - 2(200)(7) + 7^2 = 40000 - 2800 + 49 = 37249$.

3. $(7g + h)^2$.

4. $(r - 7)(r + 6)$.

5. $(x - 3)(x^2 + 3x + 9)$.

6. $(x + 2)(x^2 - 2x + 4)$.

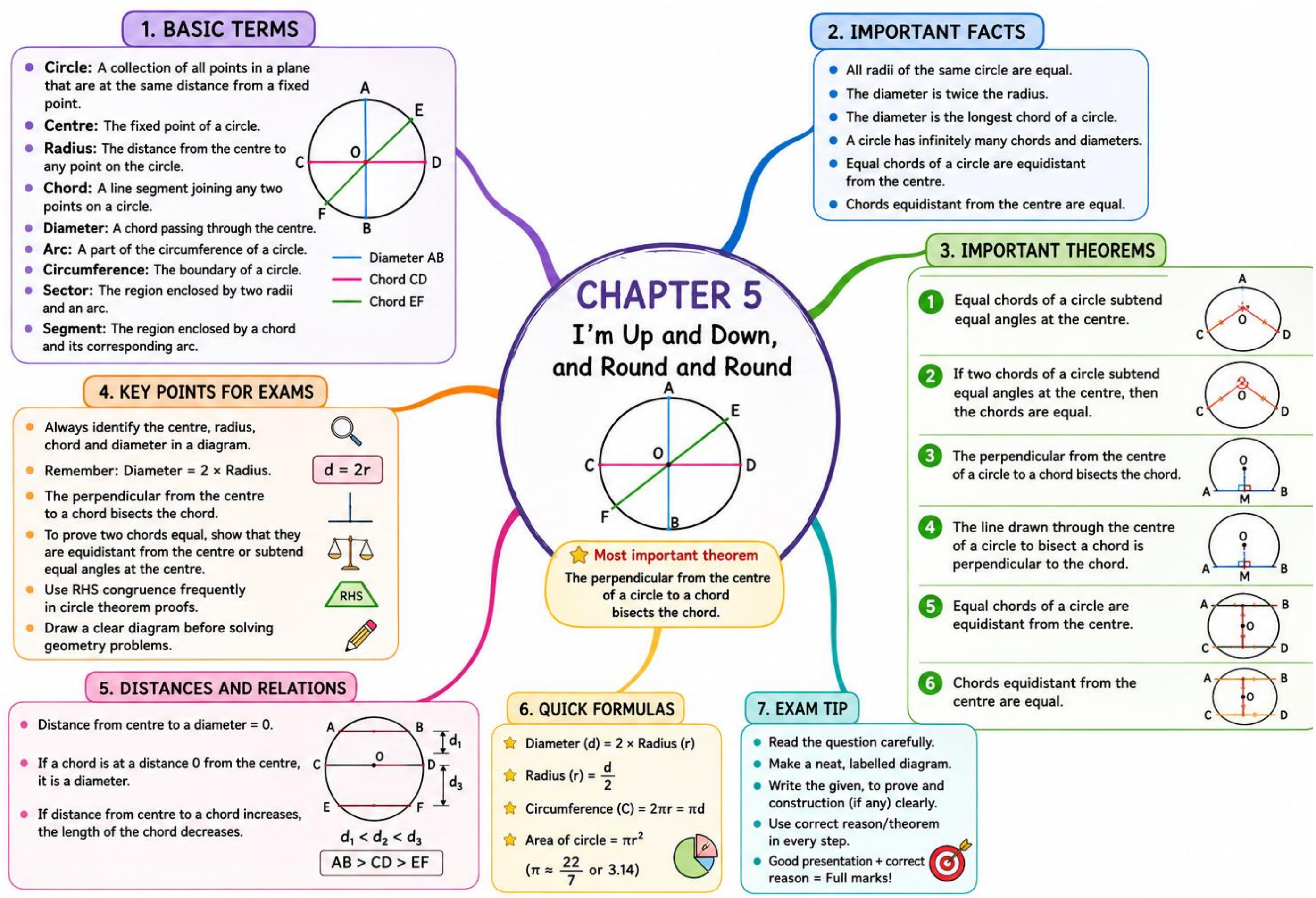
7. $8a^3 - 36a^2b + 54ab^2 - 27b^3$.

8. Side = $p + 2q$.

9. 145.

10. Outer square side = $40 + 2s$, so path area = $(40 + 2s)^2 - 40^2 = 160s + 4s^2$

Chapter 5
(I'm Up and Down, and Round and Round)



Important Points to remember:

1. Basic Terms

- Circle: A collection of all points in a plane that are at the same distance from a fixed point.
- Centre: The fixed point of a circle.
- Radius: The distance from the centre to any point on the circle.
- Chord: A line segment joining any two points on a circle.
- Diameter: A chord passing through the centre.
- Arc: A part of the circumference of a circle.
- Circumference: The boundary of a circle.
- Sector: The region enclosed by two radii and an arc.
- Segment: The region enclosed by a chord and its corresponding arc.

2. Important Facts

- All radii of the same circle are equal.
- The diameter is twice the radius.
- The diameter is the longest chord of a circle.
- A circle has infinitely many chords and diameters.
- Equal chords of a circle are equidistant from the centre.
- Chords equidistant from the centre are equal.

3. Important Theorems

Theorem 1:
Equal chords of a circle subtend equal angles at the centre.

Theorem 2:
If two chords of a circle subtend equal angles at the centre, then the chords are equal.

Theorem 3:
The perpendicular from the centre of a circle to a chord bisects the chord.

Theorem 4:
The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Theorem 5:
Equal chords of a circle are equidistant from the centre.

Theorem 6:
Chords equidistant from the centre are equal.

4. Key Points for Exams

- Always identify the centre, radius, chord and diameter in a diagram.
- Remember: Diameter = 2 × Radius.
- The perpendicular from the centre to a chord bisects the chord.
- To prove two chords equal, show that they are equidistant from the centre or subtend equal angles at the centre.
- Use RHS congruence frequently in circle theorem proofs.
- Draw a clear diagram before solving geometry problems.

5. Most important theorem to remember:
☞ *The perpendicular from the centre of a circle to a chord bisects the chord.*

Section A (1 Marks)

1. The distance from the centre of a circle to any point on the circle is called:
   A) Chord
   B) Radius
   C) Diameter
   D) Arc

   ANS- B

2. A line segment joining any two points on a circle is called:
   A) Radius
   B) Diameter
   C) Chord
   D) Secant

   ANS - C

3. The longest chord of a circle is:
   A) Radius
   B) Diameter
   C) Arc
   D) Tangent

ANS –B

4. If the radius of a circle is 7 cm, its diameter is:
   A) 3.5 cm
   B) 7 cm
   C) 14 cm
   D) 21 cm
5. Equal chords of a circle are:
   A) Unequal distances from the centre
   B) Equidistant from the centre
   C) Always diameters
   D) Always perpendicular
6. Chords equidistant from the centre of a circle are:
   A) Equal
   B) Unequal
   C) Diameters
   D) Tangents
7. The perpendicular from the centre of a circle to a chord:
   A) Bisects the chord

B) Doubles the chord
C) Is parallel to the chord
D) Has no relation to the chord

8. If a perpendicular is drawn from the centre of a circle to a chord, then the two parts of the chord are:
A) Unequal
B) Equal
C) Parallel
D) Perpendicular

9. In a circle with centre O, if chord AB is equal to chord CD, then:
A) AB and CD are equidistant from O
B) AB is a diameter
C) CD is a diameter
D) AB is perpendicular to CD

10. If two chords of a circle are at the same distance from the centre, then they are:
A) Perpendicular
B) Equal
C) Parallel
D) Diameters

11. A chord passing through the centre of a circle is called:
A) Radius
B) Arc
C) Diameter
D) Secant

12. If the diameter of a circle is 20 cm, its radius is:
A) 5 cm
B) 10 cm
C) 20 cm
D) 40 cm

13. Two circles having the same centre but different radii are called:
A) Equal circles
B) Concentric circles
C) Congruent circles
D) Intersecting circles

14. Circles having equal radii are called:
A) Concentric circles
B) Equal circles
C) Tangent circles
D) Intersecting circles

15. If two equal chords of a circle subtend angles at the centre, then these angles are:
A) Unequal
B) Equal

C) Supplementary
D) Complementary

16. In the same circle, if two chords subtend equal angles at the centre, then the chords are:
A) Equal
B) Unequal
C) Diameters
D) Perpendicular

17. A circle has radius 8 cm. The distance between the centre and a chord that is a diameter is:
A) 4 cm
B) 8 cm
C) 0 cm
D) 16 cm

18. If a chord is at a distance of 0 cm from the centre, then the chord is:
A) A radius
B) A tangent
C) A diameter
D) An arc

19. If two chords of a circle are equal, then their distances from the centre are:
A) Equal
B) Different
C) Zero
D) Cannot be determined

20. The perpendicular bisector of a chord of a circle passes through:
A) Any point on the circle
B) The centre of the circle
C) The midpoint of the radius only
D) No fixed point

ASSERTION–REASONING QUESTIONS

Directions

Choose:

A. Both Assertion and Reason are true, and Reason is the correct explanation of Assertion.
B. Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.
C. Assertion is true, but Reason is false.
D. Assertion is false, but Reason is true.

Q1. Assertion: The diameter is the longest chord of a circle.
Reason: A diameter passes through the centre of the circle.

Answer: A

Q2.Assertion: Equal chords of a circle are equidistant from the centre.
Reason: The perpendicular from the centre to a chord bisects the chord.

Answer: A

Q3. Assertion: The angle subtended by a diameter at any point on the circle is 90°.
Reason: The angle subtended by an arc at the centre is twice the angle subtended at a point on the circle.

Answer: A

Q4.Assertion: The circumcentre of an obtuse-angled triangle lies inside the triangle.
Reason: The circumcentre is the intersection of the perpendicular bisectors of the sides.

Answer: D The reason is true, but for an obtuse-angled triangle the circumcentre lies **outside** the triangle.

Q5. Assertion: Opposite angles of a cyclic quadrilateral are supplementary.
Reason: The sum of the angles subtended by the two corresponding arcs at the centre is 360°.

Answer: A

<u>SOLVED EXAMPLE</u>

1. Q1. What is the longest chord of a circle? If the radius of a circle is 7 cm, find the length of the longest chord.
   Solution:
   The longest chord of a circle is its diameter.
   Diameter = 2 × Radius
   = 2 × 7
   = 14 cm
   Answer: 14 cm

   Q2. In a circle with centre O, AB is a chord. If $OM \perp AB$, prove that AM = MB.
   Solution:
   Since the perpendicular from the centre of a circle to a chord bisects the chord,
   AM = MB
   Hence, the chord AB is bisected at M.

Q3. Two equal chords AB and CD of a circle are given. If ∠AOB = 70°, where O is the centre, find ∠COD.
Solution:
Equal chords of a circle subtend equal angles at the centre.
Therefore,
∠AOB = ∠COD
So,
∠COD = 70°

Q4. In a circle, two chords AB and CD are equal. If the distance of AB from the centre is 5 cm, find the distance of CD from the centre.
Solution:
Equal chords of a circle are equidistant from the centre.
Therefore, distance of CD from O = distance of AB from O
= 5 cm
Answer: 5 cm

Q5. If the radius of a circle is 10 cm and the distance of a chord from the centre is 6 cm, find half the length of the chord.
Solution:
Let O be the centre and OM ⊥ AB.
In right triangle OMA:
OA2=OM2+AM2
102=62+AM2
100=36+AM2
AM2=64
AM=8 cm
Therefore, half the length of the chord = 8 cm.

<u>PRACTICE QUESTIONS</u>

1. What is a chord of a circle? If the radius of a circle is 6 cm, find its diameter.
2. Prove that equal chords of a circle are equidistant from the centre.
3. In a circle with centre O, chord AB = CD. If the perpendicular distance of chord AB from O is 5 cm, find the perpendicular distance of chord CD from O.
4. Prove that the perpendicular from the centre of a circle to a chord bisects the chord.
5. In a circle of radius 10 cm, a chord is at a distance of 6 cm from the centre. Find the length of the chord.
6. Two chords of a circle are equidistant from the centre. If one chord is 12 cm long, find the length of the other chord. Give a reason.
7. In a circle with centre O, AB is a chord and OM ⊥ AB. If AB = 16 cm, find AM and MB.

8. Two equal chords AB and CD of a circle subtend angles ∠AOB and ∠COD at the centre. If ∠AOB = 70°, find ∠COD. Give a reason.
9. A circle has diameter 14 cm. Find its radius and explain why the diameter is the longest chord of the circle.
10. In a circle with centre O, chords AB and CD are at distances 4 cm and 7 cm respectively from O. Which chord is longer? Give a reason.

SOLVED EXAMPLE

Q1. Prove that equal chords of a circle are equidistant from the centre.
Answer:
Let AB and CD be two equal chords of a circle with centre O. Draw OM ⊥ AB and ON ⊥ CD.
Since the perpendicular from the centre to a chord bisects the chord:
AM=MB,CN=ND
Given:
AB=CD
Therefore,
AM=CN
In right triangles OMA and ONC:
OA=OC(radii)
and
AM=CN
Hence, by RHS congruence,
△OMA≅△ONC
Therefore,
OM=ON
Hence, equal chords are equidistant from the centre.

Q2. A chord of a circle with radius 13 cm is at a distance of 5 cm from the centre. Find the length of the chord.
Answer:
Let AB be the chord and OM ⊥ AB.
Since the perpendicular from the centre bisects the chord:
AM=MB
In right triangle OMA:
OA=13 cm,OM=5 cm
By Pythagoras theorem:
AM2=OA2−OM2
=132−52
=169−25=144
AM=12 cm
Therefore,
AB=2(12)=24 cm

Q3. Prove that equal chords of a circle subtend equal angles at the centre.
Answer:
Let AB and CD be equal chords of a circle with centre O.
Join OA, OB, OC and OD.
In triangles AOB and COD:

$OA = OC$(radii)
$OB = OD$(radii)
and
$AB = CD$(given)
Therefore,
$\triangle AOB \cong \triangle COD$
by **SSS congruence**.
Hence,
$\angle AOB = \angle COD$
Thus, equal chords subtend equal angles at the centre.

Q4. In a circle with centre O, two equal chords AB and CD subtend angles of $3x+10^\circ$ and $5x-30^\circ$ at the centre. Find x and the angles.
Answer:
Equal chords subtend equal angles at the centre.
Therefore,
$3x+10=5x-30$
$40=2x$
$x=20$
Now,
$\angle AOB=3(20)+10=70^\circ$
and
$\angle COD=5(20)-30=70^\circ$
Therefore,
$x=20, \angle AOB=\angle COD=70^\circ$

Q5. Prove that the perpendicular bisector of a chord of a circle passes through the centre of the circle.
Answer:
Let AB be a chord of a circle with centre O. Let M be the midpoint of AB.
Join OA and OB.
Since:
$OA=OB$
because both are radii, and
$AM=MB$
because M is the midpoint of AB.
Also,
$OM=OM$
Therefore,
$\triangle OMA \cong \triangle OMB$
by SSS congruence.
Hence,
$\angle OMA=\angle OMB$
These two angles form a straight angle, so each is 90°.
Thus,
$OM \perp AB$
Therefore, the perpendicular bisector of chord AB passes through **O, the centre of the circle**.

PRACTICE QUESTIONS

Q1. Prove that equal chords of a circle are equidistant from the centre.

Q2. Prove that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

Q3. In a circle with centre O, chord AB = 16 cm. The perpendicular distance of AB from O is 6 cm. Find the radius of the circle.

Q4. In a circle with centre O, two chords AB and CD are equal. If the perpendicular distance of AB from O is 5 cm, find the distance of CD from O. Give a reason.

Q5. Two chords AB and CD of a circle are at distances 4 cm and 7 cm respectively from the centre. Which chord is longer? Give a reason.

Q6. In a circle with centre O, AB and CD are two equal chords. If $OM \perp AB$ and $ON \perp CD$, prove that OM = ON.

Q7. A chord of a circle of radius 13 cm is at a distance of 5 cm from the centre. Find the length of the chord.

Q8. In a circle with centre O, chord AB is 24 cm long and $OM \perp AB$. If OM = 5 cm, find the radius of the circle.

Q9. Two equal chords AB and CD of a circle subtend angles $\angle AOB$ and $\angle COD$ at the centre. If $\angle AOB = 80°$, find $\angle COD$ and state the theorem used.

Q10. In a circle with centre O, AB is a chord and $OM \perp AB$. If AM = 9 cm and OM = 12 cm, find the radius of the circle.

SOLVED EXAMPLE

Q1. Prove that equal chords of a circle are equidistant from the centre.

Solution:

Let AB and CD be two equal chords of a circle with centre O. Draw $OM \perp AB$ and $ON \perp CD$.
Since the perpendicular from the centre of a circle to a chord bisects the chord,
$AM=MB, CN=ND$
Given,
$AB=CD$
Therefore,
$AM=CN$
Now, in right triangles OMA and ONC:
$OA=OC$ (radii)
and
$AM=CN$

Also,
$\angle OMA = \angle ONC = 90^\circ$
Therefore,
$\triangle OMA \cong \triangle ONC$
by **RHS congruence**.
Hence,
$OM = ON$
Therefore, equal chords of a circle are **equidistant from the centre**.

Q2. A chord of a circle of radius 17 cm is at a distance of 8 cm from the centre. Find the length of the chord.
Solution:
Let AB be the chord and O be the centre. Draw
$OM \perp AB$
Since the perpendicular from the centre to a chord bisects the chord,
$AM = MB$
Given:
$OA = 17$ cm, $OM = 8$ cm
In right triangle OMA, by Pythagoras theorem:
$OA^2 = OM^2 + AM^2$
$17^2 = 8^2 + AM^2$
$289 = 64 + AM^2$
$AM^2 = 225$
$AM = 15$ cm
Therefore,
$AB = 2AM$
$AB = 2(15) = 30$ cm
Answer: The length of the chord is 30 cm.

Q3. Prove that equal chords of a circle subtend equal angles at the centre.
Solution:
Let AB and CD be two equal chords of a circle with centre O.
Join OA, OB, OC and OD.
In triangles AOB and COD:
$OA = OC$
and
$OB = OD$
because all are radii of the same circle.
Also,
$AB = CD$
given.
Therefore,
$\triangle AOB \cong \triangle COD$
by **SSS congruence**.
Therefore, corresponding angles are equal:
$\angle AOB = \angle COD$
Hence, equal chords of a circle subtend **equal angles at the centre**.

Q4. In a circle with centre O, AB and CD are two equal chords. If OM $\perp$ AB and ON $\perp$ CD, prove that OM = ON.

Solution:

Since OM is perpendicular to chord AB, it bisects AB.
Therefore,
$AM=MB$
Similarly, since ON is perpendicular to chord CD,
$CN=ND$
Given:
$AB=CD$
Therefore, their halves are also equal:
$AM=CN$
Now consider right triangles OMA and ONC.
We have:
$OA=OC$
because they are radii.
Also,
$AM=CN$
and
$\angle OMA=\angle ONC=90^\circ$
Therefore,
$\triangle OMA \cong \triangle ONC$
by **RHS congruence**.
Hence, corresponding sides are equal:
$OM=ON$
Thus, equal chords are equidistant from the centre.

Q5. Two chords AB and CD of a circle are equidistant from the centre O. Prove that AB = CD.

Solution:

Draw
$OM \perp AB$
and
$ON \perp CD$
Since the chords are equidistant from the centre,
$OM=ON$
The perpendicular from the centre to a chord bisects the chord. Therefore,
$AM=MB$
and
$CN=ND$
Consider right triangles OMA and ONC.
We have:
$OA=OC$
because both are radii.
Also,
$OM=ON$
given.
And,
$\angle OMA=\angle ONC=90^\circ$
Therefore,
$\triangle OMA \cong \triangle ONC$

by **RHS congruence**.
Hence,
AM=CN
Since
AB=2AM
and
CD=2CN
we get
AB=CD
Therefore, **chords equidistant from the centre of a circle are equal**.

PRACTICE QUESTIONS

Q1.Prove that equal chords of a circle are equidistant from the centre.

Q2.Prove that the perpendicular drawn from the centre of a circle to a chord bisects the chord. Also find the length of each half of a chord of length 18 cm.

Q3.A circle has radius 13 cm. A chord is at a distance of 5 cm from the centre.

1. Find the length of the chord.
2. If another chord is equal to this chord, what is its distance from the centre? Give a reason.

Q4.In a circle with centre O, two chords AB and CD are equal. Perpendiculars OM and ON are drawn from O to AB and CD respectively.

1. Prove that **OM = ON**.
2. If OM = 6 cm and the radius of the circle is 10 cm, find the length of AB.

Q5.Two chords AB and CD of a circle are at distances 4 cm and 7 cm respectively from the centre O.

1. Which chord is longer?
2. Prove your answer using the theorem related to the distance of a chord from the centre.
3. If AB = 16 cm, explain why CD must be less than 16 cm.

CASE STUDY QUESTION

Question 1: A circular garden has centre O and radius 13 m. A chord AB is drawn at a perpendicular distance of 5 m from O. The perpendicular from O meets AB at M.

Answer the following:

(a) What is the relationship between AM and MB? (1 mark)
(b) Find the length of AM. (1 mark)
(c) Find the length of chord AB. (2 marks)

Answer Key

(a) AM = MB, because the perpendicular from the centre to a chord bisects the chord.

(b)
Using Pythagoras theorem:

$AM^2 = OA^2 - OM^2$
$= 13^2 - 5^2$
$= 169 - 25 = 144$

Therefore, AM = 12 m.

(c)
$AB = 2 \times AM$
$= 2 \times 12$
$= 24$ m.

<u>Case-Based Question 2:</u>

In a circular clock, two chords AB and CD are equal. The perpendicular distances of these chords from the centre O are represented by OM and ON respectively.

Answer the following:

(a) If AB = CD, what can you say about OM and ON? (1 mark)
(b) Which theorem supports your answer? (1 mark)
(c) If OM = 8 cm and the radius of the circle is 17 cm, find the length of chord AB. (2 marks)

Answer Key

(a) OM = ON.

(b) Equal chords of a circle are equidistant from the centre.

(c)
OA = 17 cm, OM = 8 cm.

$AM^2 = OA^2 - OM^2$
$= 17^2 - 8^2$
$= 289 - 64$
$= 225$

AM = 15 cm.

Therefore,

AB = $2 \times 15 = 30$ cm.

Case-Based Question 3

A circular park has two chords AB and CD. The distance of AB from the centre is 6 cm, while the distance of CD from the centre is 10 cm. The radius of the circle is 13 cm.

Answer the following:

(a) Which chord is nearer to the centre? (1 mark)
(b) Which chord is longer? Give a reason. (1 mark)
(c) Find the length of chord AB. (2 marks)

Answer Key

(a) Chord AB is nearer to the centre because its distance is 6 cm.

(b) AB is longer than CD, because a chord nearer to the centre of a circle is longer than a chord farther from the centre.

(c)
Radius = 13 cm
Distance of AB from centre = 6 cm.

Half of AB:

$AM^2 = 13^2 - 6^2$
$= 169 - 36$
$= 133$

$AM = \sqrt{133}$ cm

Therefore,

$AB = 2\sqrt{133}$ cm ≈ 23.07 cm.

ANSWER KEY

Answer Key

Q. No.	Answer	Q. No.	Answer
1	B	11	C
2	C	12	B
3	B	13	B
4	C	14	B
5	B	15	B
6	A	16	A
7	A	17	C
8	B	18	C
9	A	19	A
10	B	20	B

SECTION B

Answer Key

1. A chord joins any two points on a circle. Diameter = 12 cm.
2. Equal chords are equidistant from the centre — proved using congruent right triangles.
3. Distance of CD from O = 5 cm.
4. If $OM \perp AB$, then $\Delta OMA \cong \Delta OMB$ (RHS), hence AM = MB.
5. Half chord = $\sqrt{(10^2 - 6^2)} = 8$ cm. Chord = 16 cm.
6. Other chord = 12 cm, because chords equidistant from the centre are equal.
7. AM = MB = 16/2 = 8 cm each.
8. $\angle COD = 70°$, because equal chords subtend equal angles at the centre.
9. Radius = 7 cm. The diameter passes through the centre and is therefore the longest chord.
10. AB is longer, because the chord nearer to the centre is longer.

SECTION C

Answer Key

Q. No.	Answer
1	Let $OM \perp AB$ and $ON \perp CD$. Since equal chords are given, using RHS congruence, OM = ON. Hence equal chords are equidistant from the centre.
2	Let $OM \perp AB$. OA = OB (radii), OM is common and $\angle OMA = \angle OMB = 90°$. By RHS, $\Delta OMA \cong \Delta OMB$. Therefore AM = MB.
3	Half chord = 16/2 = 8 cm. Radius = $\sqrt{(8^2 + 6^2)} = \sqrt{100} = 10$ cm.
4	Distance of CD from O = 5 cm, because equal chords of a circle are equidistant from the centre.
5	AB is longer, because the chord nearer to the centre is longer.
6	OA = OC (radii), AB = CD (given), and perpendiculars bisect the chords. Thus AM = CN. By RHS congruence, OM = ON.
7	Half chord = $\sqrt{(13^2 - 5^2)} = \sqrt{144} = 12$ cm. Chord = 24 cm.
8	AM = 24/2 = 12 cm. Radius = $\sqrt{(12^2 + 5^2)} = \sqrt{169} = 13$ cm.
9	$\angle COD = 80°$. Theorem: Equal chords of a circle subtend equal angles at the centre.
10	Radius = $\sqrt{(9^2 + 12^2)} = \sqrt{225} = 15$ cm.

SECTION D

Answer Key / Solutions

Q1. Solution

Let AB and CD be equal chords of a circle with centre O. Draw perpendiculars OM and ON on AB and CD respectively.
OA = OC (radii)
AB = CD (given)
AM = AB/2 and CN = CD/2, because the perpendicular from the centre bisects a chord.
Thus, AM = CN.
In right triangles ΔOMA and ΔONC:
OA = OC
AM = CN
$\angle OMA = \angle ONC = 90°$
Therefore, $\Delta OMA \cong \Delta ONC$ by RHS congruence.
Hence, **OM = ON**.

Therefore, **equal chords of a circle are equidistant from the centre.**

Q2. Solution

Let AB be a chord and OM ⊥ AB.
In right triangles ΔOMA and ΔOMB:
OA = OB (radii)
OM = OM (common)
∠OMA = ∠OMB = 90°
Therefore, ΔOMA ≅ ΔOMB by RHS.
Hence, **AM = MB**.
Thus, the perpendicular from the centre to a chord bisects the chord.
For a chord of length 18 cm:
AM = MB = 18/2 = 9 cm.

Answer: 9 cm each.

Q3. Solution

Given:
Radius, **OA = 13 cm**
Distance of chord from centre, **OM = 5 cm**
Since OM ⊥ AB, M is the midpoint of AB.
Using Pythagoras theorem:
$OA^2 = OM^2 + AM^2$
$13^2 = 5^2 + AM^2$
$169 = 25 + AM^2$
$AM^2 = 144$
AM = 12 cm

Therefore,

AB = 2 × 12 = 24 cm.
For the second chord:
Since it is equal to AB, equal chords are equidistant from the centre.

Therefore, its distance from O is **5 cm**.

Answers:

1. Chord = **24 cm**
2. Distance = **5 cm**

Q4. Solution

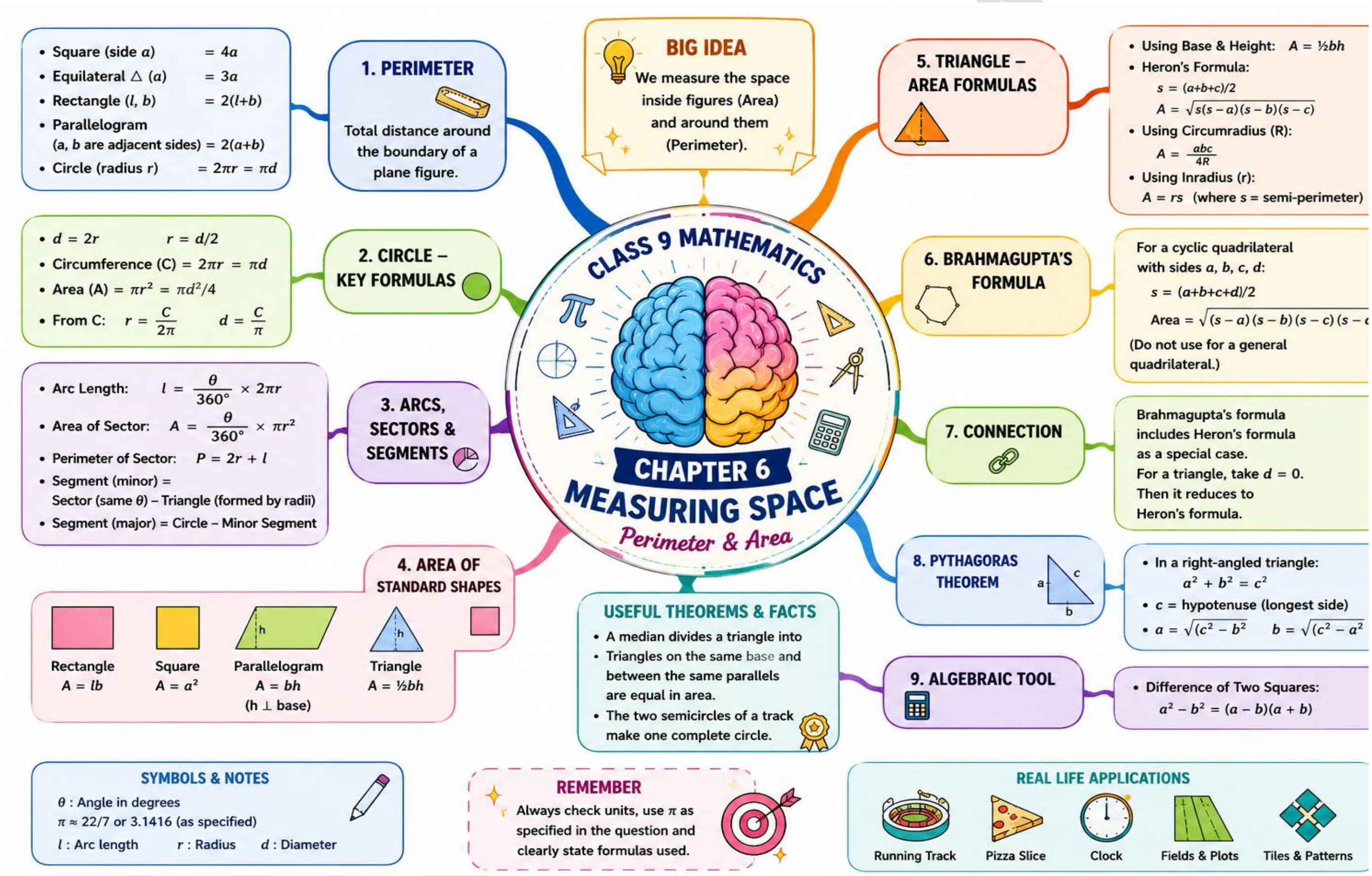
Since AB = CD, and equal chords of a circle are equidistant from the centre,
OM = ON.
Now, given:
Radius = 10 cm
OM = 6 cm
Since OM $\perp$ AB, M is the midpoint of AB.
Using Pythagoras theorem:
$OA^2 = OM^2 + AM^2$
$10^2 = 6^2 + AM^2$
$100 = 36 + AM^2$
$AM^2 = 64$
AM = 8 cm
Therefore,
AB = 2 × 8 = 16 cm.
Answers:
OM = ON
AB = 16 cm

Q5. Solution
Given:
Distance of AB from O = 4 cm
Distance of CD from O = 7 cm
A chord **nearer to the centre is longer** than a chord farther from the centre.
Since:
4 cm < 7 cm
AB is nearer to the centre than CD.
Therefore,
AB > CD.
If AB = 16 cm, then CD must be **less than 16 cm**.
Answer: AB is longer than CD.
Top of Form

CHEPTER-- 6

MEASURING SPACE: PERIMETER AND AREA

MIND MAP



CONCEPTS

1. Perimeter: Perimeter is the total length around the boundary of a shape.

2. Circle — Most Important

d = 2r   r = d/2

Circumference:

C = 2πr   C = πd

Use π ≈ 22/7 ≈ 3.14 when the question specifies it. π is irrational.

$r = C/(2\pi)$ $d = C/\pi$

3. Arc Length: If an arc subtends θ° at the centre:

$L = (\theta/360°) \times 2\pi r$

(Arc length = fraction of the circle × circumference)

Semicircle: $L = \pi r$ Quarter circle: $L = \pi r/2$

Perimeter of a sector:

$P = 2r + \text{arc length} = 2r + (\theta/360°) \times 2\pi r$

4. Area of Standard Shapes

Rectangle: $A = lb$ Square: $A = a^2$

Parallelogram: $A = bh$ Triangle: $A = \frac{1}{2}bh$

For a parallelogram, h is the perpendicular height, not necessarily the slanting side.

5. Important Triangle Property — Median: A median joins a vertex to the midpoint of the opposite side.

If AD is a median of △ABC: BD = DC, and Area(△ABD) = Area(△ACD)

A median divides a triangle into two triangles of equal area.

6. Heron's Formula: Use this when all three sides of a triangle are known but the height is not.

$s = (a+b+c)/2$ $A = \sqrt{s(s-a)(s-b)(s-c)}$

7. Triangle Area Using Incircle / Circumcircle

Using circumradius R: $A = abc/(4R)$

Using inradius r: $A = rs$, equivalently $A = r(a+b+c)/2$

8. Area of a Circle

$A = \pi r^2 = \pi d^2/4 = \frac{1}{2}Cr$

9. Sector of a Circle: A sector is the region bounded by two radii and an arc.

Area of sector = $(\theta/360°) \times \pi r^2$

Semicircle: $\frac{1}{2}\pi r^2$ Quarter circle: $\frac{1}{4}\pi r^2$

10. Segment of a Circle: A segment is the region bounded by an arc and the chord joining the endpoints of the arc.

Minor segment = minor sector − triangle Major segment = circle − minor segment

11. Brahmagupta's Formula: Used for a cyclic quadrilateral when its four sides are known.

$s = (a+b+c+d)/2 \qquad A = \sqrt{[(s-a)(s-b)(s-c)(s-d)]}$

Do not use Brahmagupta's formula for an arbitrary quadrilateral; it applies to a cyclic quadrilateral.

12. Brahmagupta ↔ Heron Connection: Brahmagupta's formula generalises Heron's formula. With the fourth side taken as zero, it reduces to Heron's formula.

13. Difference of Two Squares

$a^2 - b^2 = (a-b)(a+b)$

14. Athletics Track Concept: For a track with two straight sections and two semicircles, the two semicircles together form one complete circle.

Total distance = straight portions + 2πr

Outer lanes have longer curved portions because their radii are larger.

SOLVED QUESTIONS

1. A circular park has diameter 28 m.

(a) Find its circumference using $\pi = 22/7$.

(b) If the diameter is doubled, what happens to the circumference?

(c) Is $\pi = 22/7$ exactly? Explain.

Solution

(a) $C = \pi d = (22/7) \times 28 = 88$ m. Therefore, $C = 88$ m.

(b) If the diameter changes from d to 2d: $C' = \pi(2d) = 2\pi d = 2C$. Therefore, the circumference also doubles.

(c) No. $\pi \neq 22/7$. The value 22/7 is an approximation to π.

2. A 400 m athletics track consists of two straight portions of 84.39 m each and two semicircular portions. The radius of the innermost semicircle is 36.5 m. Each lane is 1.22 m wide.

A runner in the first lane runs 0.3 m from the inner boundary.

Find the additional distance travelled on the curved portions by a runner in the second lane compared with the first runner. Take $\pi = 3.1416$.

Solution

The first runner's radius is: $r_1 = 36.5 + 0.3 = 36.8$ m.

The second lane is 1.22 m farther out: $r_2 = 36.8 + 1.22 = 38.02$ m.

The two semicircles together form a complete circle. Therefore, the difference in curved distance is: $2\pi r_2 - 2\pi r_1 = 2\pi(r_2 - r_1) = 2(3.1416)(1.22) \approx 7.6655$ m.

Answer: approximately 7.67 m. Insight: the straight portions are the same for both runners; the difference comes from the larger radius on the curved portions.

3. A sector of a circle has radius 14 cm and central angle 75°. Find: (i) the length of its arc (ii) the perimeter of the sector. Use $\pi = 22/7$.

Solution

(i) Arc length: $l = (75/360) \times 2\pi r = (75/360) \times 2 \times (22/7) \times 14 = 55/3 = 18\frac{1}{3}$ cm.

(ii) Perimeter of a sector = arc + two radii: $P = 55/3 + 2(14) = 55/3 + 84/3 = 139/3 = 46\frac{1}{3}$ cm.

Answer: arc = $18\frac{1}{3}$ cm; perimeter = $46\frac{1}{3}$ cm.

4.The sides of a triangle are 13 cm, 14 cm and 15 cm. Find its area using Heron's formula.

Solution

Given $a = 13$, $b = 14$, $c = 15$. $s = (13 + 14 + 15)/2 = 21$.

By Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84$ cm².

Answer: 84 cm².

[**HOTS**]

5. A rectangle has perimeter 40 cm. (i) Express its area in terms of one side x. (ii) Find the dimensions for which its area is maximum. (iii) Find the maximum possible area.

Solution

Let the sides be x and y. $2(x+y) = 40 \Rightarrow x+y = 20 \Rightarrow y = 20-x$.

(i) $A = xy = x(20-x) = 20x-x^2$. Complete the square: $A = 100 - (x-10)^2$. Since $(x-10)^2 \geq 0$, $A \leq 100$.
Maximum occurs when $x = 10$, then $y = 10$.

(ii) Dimensions = 10 cm × 10 cm. (iii) Maximum area = 100 cm^2.
Conclusion: among rectangles with a fixed perimeter, the square has the maximum area.

6. A circle has radius 15 cm. A chord subtends 60° at the centre. Using $\pi = 3.14$ and $\sqrt{3} = 1.73$, find: (i) area of the minor sector (ii) area of the triangle formed by the two radii and the chord (iii) area of the minor segment (iv) area of the major segment.

Solution

Minor sector: $A = (60/360)\pi r^2 = (1/6)(3.14)(225) = 117.75$ cm^2.

(ii) The two radii and chord form an equilateral triangle because all three sides are 15 cm. Height = $(\sqrt{3}/2)(15) = (1.73/2)(15) = 12.975$ cm. Triangle area = $\frac{1}{2}(15)(12.975) = 97.3125$ cm^2.

(iii) Minor segment = minor sector − triangle = $117.75 - 97.3125 = 20.4375$ cm^2.

(iv) Circle area = $3.14(225) = 706.5$ cm^2. Major segment = circle − minor segment = $706.5 - 20.4375 = 686.0625$ cm^2.

Key distinction: a sector is bounded by two radii and an arc; a segment is bounded by a chord and an arc.

7. Two congruent circles have radius r. The centre of each circle lies on the circumference of the other. Find the perimeter of the boundary formed by the outer arcs of the two circles in terms of r.

Solution

Let the centers be A and B, and the intersection points be C and D. Since each circle passes through the other's centre: AB = AC = BC = r.

Thus ΔABC is equilateral, so the relevant central angle is 60°. The outer arc on each circle is therefore: $360° - 2(60°) = 240°$.

Length of one outer arc: $(240/360)(2\pi r) = 4\pi r/3$. There are two such arcs: $P = 2(4\pi r/3) = 8\pi r/3$.

Answer: $P = 8\pi r/3$.

8. A cyclic quadrilateral is actually a rectangle with sides 8 cm and 15 cm. Use Brahmagupta's formula to find its area and verify that the result agrees with the ordinary rectangle formula.

Solution

The four sides are 8, 15, 8 and 15 cm. $s = (8+15+8+15)/2 = 23$.
Brahmagupta: $A = \sqrt{[(23-8)(23-15)(23-8)(23-15)]} = \sqrt{[15\times8\times15\times8]} = \sqrt{(120^2)} = 120\ cm^2$.
Verification using rectangle formula: $A = l\times b = 15\times8 = 120\ cm^2$. Both methods give the same answer.
Reason: every rectangle is a cyclic quadrilateral, so Brahmagupta's formula applies.

SECTION A

1. The perimeter of a rectangle of length a units and width b units is:

a) a+b
b) 2(a+b)
c) ab
d) 2ab

2. For all circles, the ratio of circumference C to diameter D is:

a) variable with radius
b) always 2
c) a constant called π
d) always 4

3. A circle has diameter 28 cm. Using π = 22/7, its circumference is:

a) 44 cm
b) 88 cm
c) 176 cm
d) 616 cm

4. If the side of a square is doubled, its perimeter becomes:

a) half
b) unchanged
c) double
d) four times

5. Which statement about π is correct according to the chapter?

a) π is exactly 22/7
b) π is a rational number
c) π is an irrational number
d) π depends on the size of the circle

6. The arc length corresponding to a central angle θ in a circle of radius r is:

a) $(\theta/180°)\pi r^2$
b) $(\theta/360°)2\pi r$
c) $2\pi r\theta$
d) $\pi r^2/\theta$

7. A sector has radius 14 cm and angle 90°. Using π = 22/7, its arc length is:

a) 11 cm
b) 22 cm
c) 44 cm
d) 88 cm

8. The area of a parallelogram with base b and height h is:

a) 2bh
b) bh
c) ½bh
d) b+h

9. A triangle has base 12 cm and perpendicular height 5 cm. Its area is:

a) 17 cm²
b) 30 cm²
c) 60 cm²
d) 120 cm²

10. A median of a triangle divides it into two triangles having:

a) equal perimeters
b) equal bases
c) equal areas
d) equal angles

11. For a triangle with sides a, b and c, the semi-perimeter used in Heron's formula is:

a) a+b+c
b) (a+b+c)/2
c) 2(a+b+c)
d) abc/2

12. A triangle has sides 3 cm, 4 cm and 5 cm. By Heron's formula, its area is:

a) 5 cm^2
b) 6 cm^2
c) 10 cm^2
d) 12 cm^2

13. Which pair is correctly matched?

a) Incircle — passes through all three vertices
b) Circumcircle — touches all three sides
c) Circumcircle — passes through the three vertices
d) In circle — has centre outside the triangle

14. A segment of a circle is the region bounded by:

a) two radii and an arc
b) a chord and an arc
c) two chords
d) a diameter and two radii only

15. The area of a sector of radius r and central angle θ is:

a) $(\theta/360°)\pi r^2$
b) $(\theta/360°)2\pi r$
c) $\pi r/\theta$
d) $2\pi r^2$

16. A circle has radius 7 cm. The area of a 60° sector, using π = 22/7, is:

a) 77/3 cm^2

b) 154/3 cm²
c) 44 cm²
d) 77 cm²

17. Which statement about Brahmagupta's formula is correct?

a) It applies only to rectangles
b) It gives the area of any triangle directly from its angles
c) It gives the area of a cyclic quadrilateral from its four sides
d) It is unrelated to Heron's formula

18. If all sides of a triangle are multiplied by 3, its area becomes:

a) 3 times
b) 6 times
c) 9 times
d) unchanged

<u>ASSERTION AND REASON QUESTIONS</u>

(a) Both Assertion(A) and Reason(R) are true and the Reason(R) is the correct explanation of Assertion(A).

(b) Both Assertion(A) and Reason(R) are true but the Reason(R) is not the correct explanation of Assertion(A).

(c) Assertion(A) is true, but Reason(R) is false.

(d) Assertion(A) is false, but Reason(R) is true.

19. Assertion (A): If the diameter of a circle is doubled, its circumference is doubled.

Reason (R): The circumference of a circle is $C = \pi d$.

20. Assertion (A): Heron's formula can be obtained as a special case of Brahmagupta's formula.

Reason (R): A triangle can be regarded as a cyclic quadrilateral with its fourth side having zero length.

<u>SECTION B</u>

21. The perimeter of a circle is 44 cm. Using $\pi = 22/7$, find its radius.

22.A circle has radius 7 cm. Find the length of the arc subtending 60° at the centre. Use $\pi = 22/7$.

23. What is the area of a parallelogram whose base is 12 cm and perpendicular height is 5 cm?

24.A triangle has sides 3 cm, 4 cm and 5 cm. What is its area?

25. State the formula for the area of a sector of radius r subtending an angle θ° at the centre.

SECTION C

26.A circular field has circumference 88 m. Find its radius and area, taking π = 22/7.

27. Find the length of an arc of radius 6.3 m subtending an angle of 120° at the centre. Take π = 22/7.

28. The base and perpendicular height of a parallelogram are 18 cm and 7 cm respectively. Find its area.

29. The sides of a triangle are 5 cm, 12 cm and 13 cm. Find its area using Heron's formula.

30. State the difference between a sector and a segment of a circle.

SECTION D

31.A triangular plot has sides 13 m, 14 m and 15 m. Find its area using Heron's formula. If the cost of levelling the plot is ₹25 per square metre, find the total cost.

32.A sector of a circle has radius 14 cm and central angle 75°. Find (i) its arc length and (ii) its perimeter. Use π = 22/7.

33.A chord of a circle of radius 10 cm subtends 90° at the centre. Find the areas of (i) the minor sector and (ii) the major sector. Take π = 3.14.

34.A cyclic quadrilateral has sides 5 cm, 6 cm, 7 cm and 8 cm. Find its area using Brahmagupta's formula. Then explain how Heron's formula appears as a special case.

Case study

35.A school athletics track has two straight portions and two semicircular portions. The two semicircles together make one complete circle. A runner in an outer lane travels farther on the curved portions because the radius is larger.

(a) Write the formula for the circumference of a circle.

(b) If the radius of one lane is 36.8 m, find the curved distance for the two semicircles, taking π = 3.1416.

(c) If the next lane is 1.22 m farther out, find the additional curved distance travelled.

(d) Why is a stagger necessary on an athletics track?

36. The minute hand of a clock is 7 cm long. During a period of 10 minutes, it sweeps through a sector of the circular region.

(a) Through what angle does the minute hand turn in 10 minutes?

(b) Find the area swept by the minute hand, taking π = 22/7.

(c) State the formula used for the area of a sector.

(d) What fraction of the full circular area is swept?

37. A triangular plot has side lengths 8 m, 11 m and 13 m. The owner wants to know its area before planning the plot.

(a) Find its semi-perimeter.

(b) Which formula is appropriate when all three sides are known?

(c) Find the area.

(d) If the plot is to be covered at ₹50 per square meter, find the approximate cost.

38.A car has two non-overlapping wipers. Each wiper blade has length 28 cm and sweeps through 120°. Each sweep cleans a sector-shaped region.

(a) What is the radius of each swept sector?

(b) Find the area cleaned by one wiper in one sweep, taking π = 22/7.

(c) Find the total area cleaned by both wipers.

(d) Is the swept region a sector or a segment? Give a reason.

ANSWER KEY

Section A (MCQs)

1. (b) $2(a+b)$

2. (c) a constant called π

3. (b) 88 cm

4. (c) double

5. (c) π is an irrational number

6. (b) $(\theta/360°)2\pi r$

7. (b) 22 cm

8. (b) bh

9. (b) 30 cm^2

10. (c) equal areas

11. (b) $(a+b+c)/2$

12. (b) 6 cm^2

13. (c) Circumcircle — passes through the three vertices

14. (b) a chord and an arc

15. (a) $(\theta/360°)\pi r^2$

16. (a) 77/3 cm^2

17. (c) It gives the area of a cyclic quadrilateral from its four sides

18. (c) 9 times

Assertion and Reason Questions

19. (a) Both Assertion(A) and Reason(R) are true and the Reason(R) is the correct explanation of Assertion(A).

20. (a) Both Assertion(A) and Reason(R) are true and the Reason(R) is the correct explanation of Assertion(A).

Section B

21. Solution: $C = 2\pi r \Rightarrow 44 = 2 \times (22/7) \times r \Rightarrow r = 7$ cm. Answer: 7 cm.

22. Solution: Arc length $= (\theta/360°) \times 2\pi r = (60/360) \times 2 \times (22/7) \times 7 = 22/3$ cm. Answer: 22/3 cm (≈ 7.33 cm).

23. Solution: Area of parallelogram = base × height $= 12 \times 5 = 60$ cm². Answer: 60 cm².

24. Solution: Using Heron's formula: $s = (3+4+5)/2 = 6$. Area $= \sqrt{[6(6-3)(6-4)(6-5)]} = \sqrt{(6\times3\times2\times1)} = \sqrt{36} = 6$ cm². Answer: 6 cm².

25. Solution: Area of sector $= (\theta/360°) \times \pi r^2$. Answer: $A = (\theta/360°)\pi r^2$.

Section C

26. Solution: $C = 2\pi r \Rightarrow 88 = 2 \times (22/7) \times r \Rightarrow r = 14$ m. Area $= \pi r^2 = (22/7) \times 14^2 = 616$ m². Answer: radius = 14 m; area = 616 m².

27. Solution: Arc length $= (\theta/360°) \times 2\pi r = (120/360) \times 2 \times (22/7) \times 6.3 = 13.2$ m. Answer: 13.2 m.

28. Solution: Area of parallelogram = base × height $= 18 \times 7 = 126$ cm². Answer: 126 cm².

29. Solution: $s = (5+12+13)/2 = 15$. Area $= \sqrt{[s(s-a)(s-b)(s-c)]} = \sqrt{[15(10)(3)(2)]} = \sqrt{900} = 30$ cm². Answer: 30 cm².

30. Solution: A sector is the region bounded by two radii and the corresponding arc. A segment is the region bounded by a chord and the corresponding arc. Answer: sector → two radii + arc; segment → chord + arc.

Section D

31. Given a = 13 m, b = 14 m, c = 15 m. $s = (13+14+15)/2 = 21$ m. By Heron's formula: Area $= \sqrt{[21(21-13)(21-14)(21-15)]} = \sqrt{(21\times8\times7\times6)} = \sqrt{7056} = 84$ m². Cost = 84 × ₹25 = ₹2,100. Answer: area = 84 m²; total levelling cost = ₹2,100.

32. (i) Arc length: $l = (75/360) \times 2\pi r = (75/360) \times 2 \times (22/7) \times 14 = 55/3$ cm $= 18\frac{1}{3}$ cm. (ii) Perimeter of a sector = arc + two radii = 55/3 + 28 = 139/3 cm $= 46\frac{1}{3}$ cm. Answer: arc length $= 18\frac{1}{3}$ cm; perimeter $= 46\frac{1}{3}$ cm.

33. Total area of circle $= \pi r^2 = 3.14 \times 100 = 314$ cm². (i) Minor sector: Area = (90/360) × 314 = 78.5 cm². (ii) Major sector: Angle = 360° − 90° = 270°. Area = (270/360) × 314 = 235.5 cm². Answer: minor sector = 78.5 cm²; major sector = 235.5 cm².

34. For the cyclic quadrilateral: a=5, b=6, c=7, d=8. s = (5+6+7+8)/2 = 13. Brahmagupta's formula: Area $= \sqrt{[(s-a)(s-b)(s-c)(s-d)]} = \sqrt{(8\times7\times6\times5)} = \sqrt{1680} = 4\sqrt{105}$ cm² ≈ 40.99 cm². For a triangle, take the fourth side d = 0. Then $s = (a+b+c)/2$, and Brahmagupta becomes $\sqrt{[(s-a)(s-b)(s-c)s]}$, which is Heron's formula. Answer: area $= 4\sqrt{105}$ cm² ≈ 40.99 cm²;

Heron's formula is the special case d = 0.

Case Study

35. (a) $C = 2\pi r$. (b) The two semicircles form a complete circle: C = 2(3.1416)(36.8) ≈ 231.22 m. (c) New radius = 36.8 + 1.22 = 38.02 m. Additional distance = $2\pi(1.22)$ = 7.6655 m ≈ 7.67 m. (d) The outer lane has a larger radius, so its curved path is longer. A stagger ensures that runners cover equal total race distances.

36. (a) In 60 minutes the hand turns 360°. In 10 minutes: $\theta = (10/60) \times 360° = 60°$. (b) Area = (60/360) × (22/7) × 7² = 77/3 cm² ≈ 25.67 cm². (c) Sector area $= (\theta/360°)\pi r^2$. (d) The fraction is 60/360 = 1/6.

37. (a) s = (8+11+13)/2 = 16 m. (b) Heron's formula. (c) Area $= \sqrt{[16(16-8)(16-11)(16-13)]} = \sqrt{(16\times8\times5\times3)} = \sqrt{1920} = 8\sqrt{30}$ m² ≈ 43.82 m². (d) Cost ≈ 43.82 × ₹50 ≈ ₹2,191.

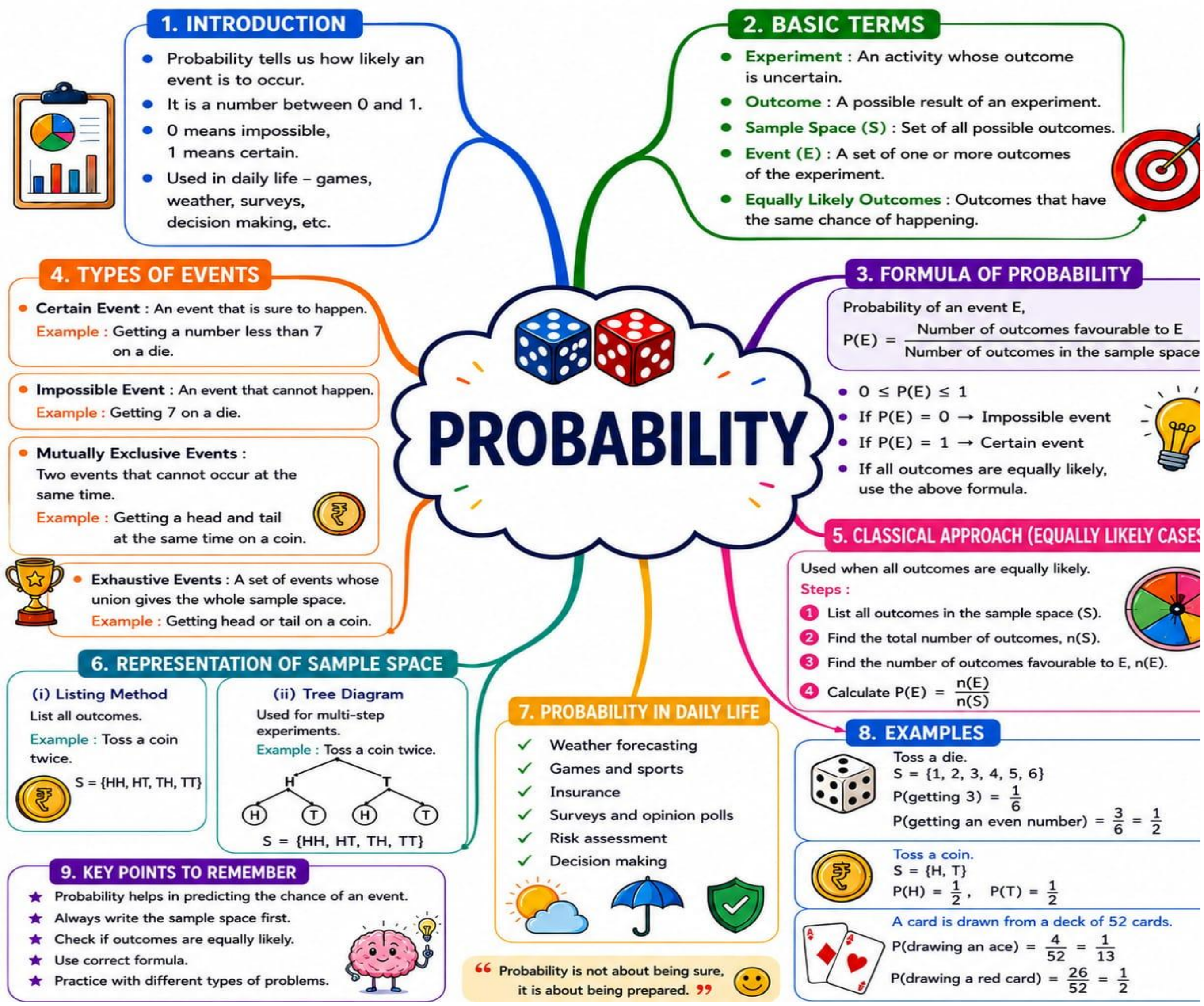
38. (a) Radius = 28 cm. (b) Area of one sector = (120/360) × (22/7) × 28² = (1/3) × (22/7) × 784 $= 821\frac{1}{3}$ cm². (c) Two wipers do not overlap: Total area = 2 × $821\frac{1}{3} = 1642\frac{2}{3}$ cm² ≈ 1642.67 cm². (d) It is a sector because the region is bounded by two radii (the positions of the blade) and the arc swept by its end.

CHAPTER 7

The Mathematics of Maybe

Introduction to Probability

MIND MAP



What is Probability?

Probability is a way of measuring how likely something is to happen, just as length, area, or volume measure physical quantities.

Example 1 — Subjective Probability

One friend says, “The sun is shining brightly, so it's unlikely to rain today,” while another says, “It's very hot, so it could rain later.”

Both friends are forming a subjective probability — an estimate based on personal interpretation of evidence, rather than on data or calculation.

What is Randomness?

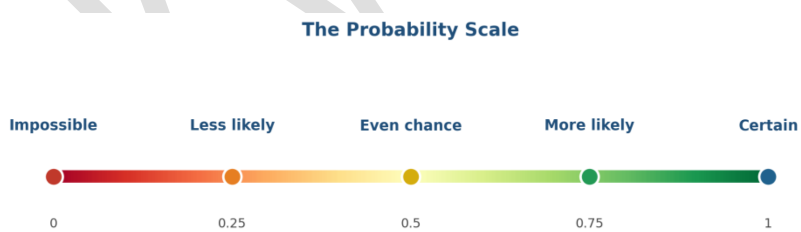
Randomness describes a situation where you know all the possible outcomes but cannot predict exactly which one will happen.

Example 2 — Recognising Randomness

Which of these are random experiments? (a) Tossing a fair coin. (b) Adding $2 + 2$. (c) Rolling a die. (d) Boiling water at sea level.

Answer: (a) and (c) are random — the outcome is uncertain in advance. (b) and (d) are not random — they always give the same, predictable result.

The Probability Scale



7.2 Measuring Probability Objectively

There are two main ways to estimate probability objectively:

✓Experimental probability — based on evidence collected from actually performing an experiment or from past data.

✓Theoretical probability — based on reasoning, assuming all outcomes are equally likely in an ideal, fair situation.

7.2.1 Experimental Probability

In an experiment, a result is called an outcome, and the set of all possible outcomes is called the sample space.

Experimental Probability = (Number of times the event occurred) ÷ (Total number of trials)

Example 4 — Rolling a Die 50 Times

A die is rolled 50 times and lands on 4 exactly 8 times. Find the experimental probability of rolling a 4.

Experimental Probability = 8 ÷ 50 = 0.16, or 16%.

✎ Example 5 — Free-Throw Shooting

A basketball player attempts 40 free throws in practice and scores 28 of them. Estimate the probability that her next free throw is successful.

Experimental Probability = 28 ÷ 40 = 0.7, or 70%.

7.2.2 Theoretical Probability

Theoretical probability studies the likelihood of an event assuming a perfectly fair situation, where every outcome is equally likely. No experiment or data collection is needed — only reasoning about outcomes.

Theoretical Probability, P(A) = (Number of favourable outcomes) ÷ (Number of possible outcomes)

✎ Example 6 — Rolling a 4 on a Die

What is the theoretical probability of rolling a 4 on a standard 6-sided die?

Favourable outcomes = 1 (only the number 4); Possible outcomes = 6

P(rolling a 4) = 1/6 ≈ 0.167, or 16.7%.

✎ Example 7 — The Letter B in 'PROBABILITY'

A letter is picked at random from the word 'PROBABILITY'. What is the probability of picking the letter B?

Favourable outcomes = 2 (there are two Bs); Possible outcomes = 11 (total letters)

P(picking B) = 2/11 ≈ 0.182, or 18.2%.

7.2.3 Analysing Statistical Data Using Probability

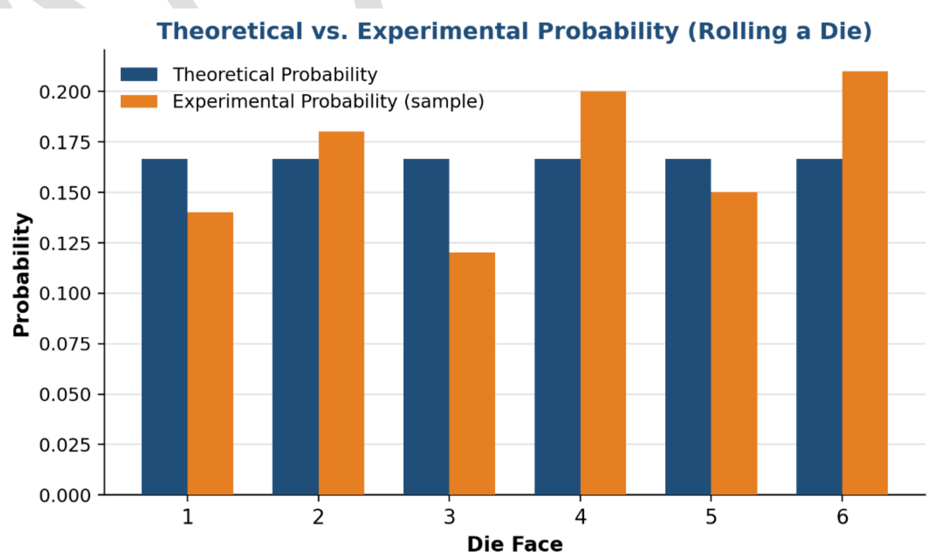
Statistical probability, drawn from real survey or business data, is widely used in marketing, sales forecasting, insurance, and research.

✎ Example 11 — Quality Control

A factory tests 200 light bulbs from a batch and finds 6 are defective. Estimate the probability that a randomly chosen bulb from this batch is defective.

P(defective) = 6/200 = 0.03, or 3%. In a shipment of 5000 bulbs, we would expect about 150 to be defective.

Experimental probability is based on actual data from trials; theoretical probability is based on the assumption of equally likely outcomes and needs no data.



⚠ Gambler's Fallacy

A common misunderstanding is believing that if an outcome (e.g., heads) has occurred many times in a row, the opposite outcome (tails) becomes "due" to happen next. In reality, each independent trial — like a coin toss or die roll — has no memory of past results. If a fair coin has landed heads six times in a row, the probability of tails on the next toss is still exactly 0.5. Probability predicts long-run patterns, not what happens on the very next trial.

7.3 Elements of Probability: Sample Spaces and Events

7.3.1 Sample Space

The sample space (denoted S) is the complete list of all possible outcomes of an experiment; each outcome is an element of S. It must include every possible outcome exactly once. The number of elements in S is called the sample size, written n(S).

✓Tossing a coin: S = {Heads, Tails}, n(S) = 2

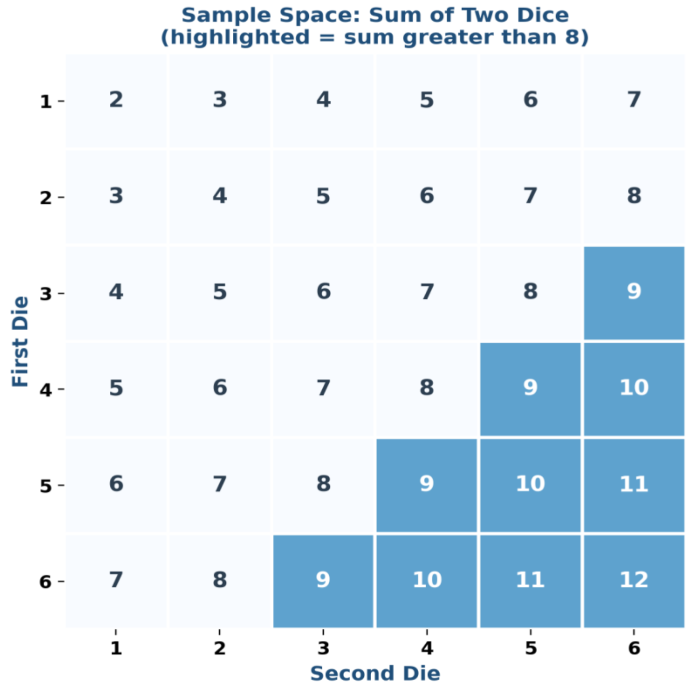
✓Rolling a die: S = {1, 2, 3, 4, 5, 6}, n(S) = 6

Example 12 — Sum of Two Dice

Two dice are rolled together and their sum is recorded. What is the probability that the sum is greater than 8?

Total possible outcomes = 6 × 6 = 36. Outcomes with sum > 8 are highlighted below — there are 10 of them.

P(sum > 8) = 10/36 = 5/18 ≈ 0.278, or 27.8%.



Sample space for the sum of two dice; shaded cells show sums greater than 8.

Events

An event is any single outcome or combination of outcomes from a random experiment — that is, any subset of the sample space.

Experiment	Sample Space	Example Event
Tossing two coins	{HH, HT, TH, TT}	At least one Head: {HH, HT, TH}
Rolling a 6-sided die	{1, 2, 3, 4, 5, 6}	Number greater than 4: {5, 6}
Picking fruit from a basket	{Apple, Banana, Orange}	Picking a yellow fruit: {Banana}

✎ Example 13 — Identifying an Event

A single card is drawn from a 6-card set numbered 1 to 6. Let event E be 'drawing an even number'. List E and find P(E).

S = {1, 2, 3, 4, 5, 6}; E = {2, 4, 6}; P(E) = 3/6 = 0.5, or 50%.

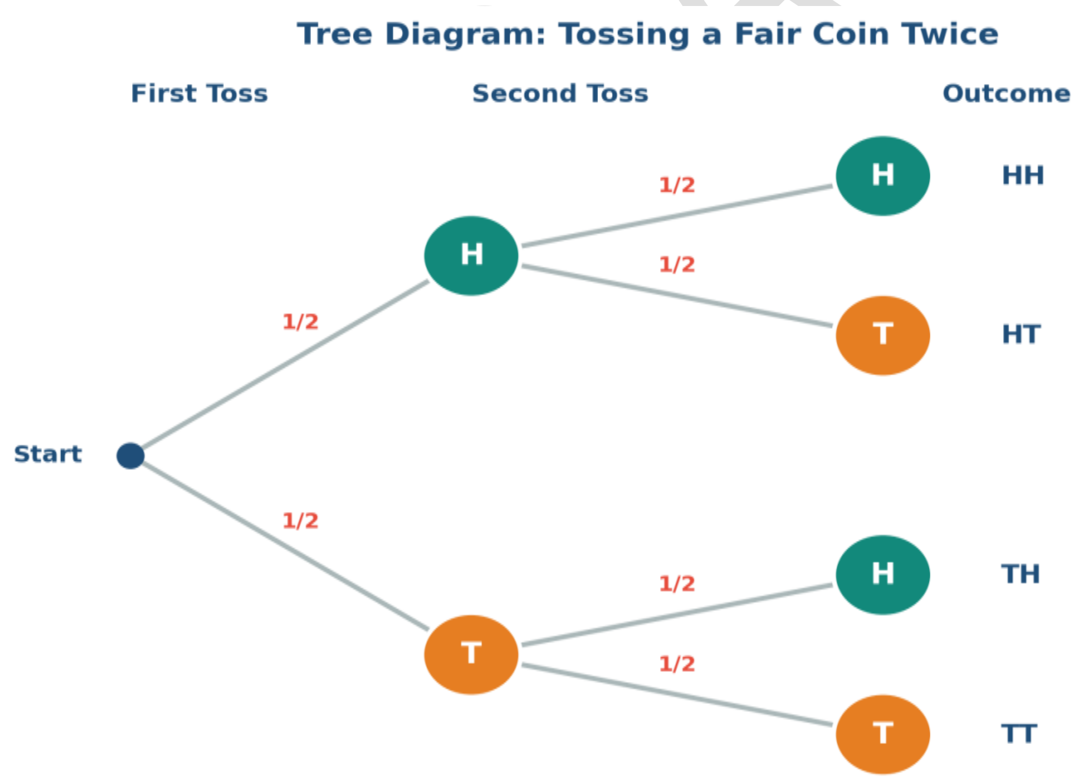
7.4 Tree Diagrams

A tree diagram is a visual way of listing all possible outcomes of a multi-step experiment .Each branch represents a possible outcome, splitting further to show subsequent events; every complete path from start to end represents one outcome in the sample space.

✎ Example 14 — Tossing a Coin Twice

Draw a tree diagram for tossing a fair coin twice, and find the probability of getting two heads.

Sample space: S = {HH, HT, TH, TT}. Multiplying along a path: P(HH) = $1/2 \times 1/2 = 1/4 = 0.25$, or 25%.



EXAMPLES

Section A: Multiple Choice Questions

1. A bag contains only blue marbles. What is the probability of drawing a red marble from it?

   (A) 0 (B) 0.5

   (C) 1(D) Cannot be determined

2. If P(E) = 0.35, what is the probability of the event 'not E'?

   (A) 0.35 (B) 0.65 (C) 1.3 (D) 0

3. A weather forecaster says there is a 20% chance of rain tomorrow. This value is best classified as:

   (A) Theoretical probability (B) Statistical/experimental probability

   (C) An impossible event (D) A certain event

4. Two coins are tossed together. What is n(S), the size of the sample space?

   (A) 2 (B) 3 (C) 4 (D) 8

5. A die is rolled 60 times and a '5' appears 12 times. The experimental probability of rolling a '5' is:

   (A) 1/6 (B) 1/5 (C) 12/60 = 0.2 (D) 5/60

6. Which of the following CANNOT be a value of P(E)?

   (A) 0 (B) 0.75 (C) 1 (D) 1.2

7. A coin is tossed 5 times and lands heads every time. According to correct probability reasoning, the probability of heads on the 6th toss is:

   (A) Less than 0.5, since heads is 'due' for a change

   (B) Greater than 0.5, since heads is 'on a streak'

   (C) Still 0.5, as each toss is independent

   (D) 0, since heads cannot occur 6 times in a row

8. As the number of trials in an experiment increases, the experimental probability tends to approach the theoretical probability. This principle is known as:

(A) Gambler's Fallacy (B) Law of Large Numbers

(C) Sampling Bias (D) Relative Frequency Rule

9. A letter is chosen at random from the word 'CERTAIN'. What is the probability that it is a vowel?

(A) 2/7 (B) 3/7 (C) 4/7 (D) 1/7

10. A spinner is divided into 5 equal sectors numbered 1 to 5. What is the probability of NOT landing on 3?

(A) 1/5 (B) 2/5 (C) 3/5 (D) 4/5

ANSWERS

1. (A) 0 — the bag has no red marbles, so drawing one is impossible.

2. (B) 0.65 — since P(not E) = 1 − P(E) = 1 − 0.35 = 0.65.

3. (B) Statistical/experimental probability — based on collected weather data/patterns.

4. (C) 4 — Sample space = {HH, HT, TH, TT}.

5. (C) 12/60 = 0.2 — experimental probability = favourable trials ÷ total trials.

6. (D) 1.2 — probability must lie between 0 and 1 (inclusive).

7. (C) Still 0.5 — each coin toss is an independent event (Gambler's Fallacy warns against A/B).

8. (B) Law of Large Numbers.

9. (B) 3/7 — CERTAIN has 7 letters, with 3 vowels: E, A, I.

10. (D) 4/5 — P(not 3) = 1 − 1/5 = 4/5.

Section B: Very Short Answer Questions

1. What is the probability of an event that is certain to happen?

SOLUTION- 1 (certainty)

2. A die is rolled once. Write the sample space.

SOLUTION- S = {1, 2, 3, 4, 5, 6}.

3.If a bag has 4 red and 6 green balls, what is n(S) when one ball is drawn?

SOLUTION- n(S) = 10 (4 red + 6 green balls).

Section C: Short Answer Questions (2–3 marks each)

1. A survey of 60 people found that 18 preferred tea over coffee. Find the experimental probability that a randomly chosen person from this group prefers tea. If the town has 9000 people, estimate how many prefer tea.

SOLUTION Experimental probability = 18/60 = 0.3

Estimated number in town = 0.3 × 9000 = 2700 people.

2. A card is drawn at random from a well-shuffled pack of 52 playing cards. Find the probability that the card drawn is a king.

SOLUTION There are 4 kings in a deck of 52 cards, so

P(king) = 4/52 = 1/13 ≈ 0.077.

3. A bag contains 3 red, 2 white and 5 black balls. A ball is drawn at random. Find the probability that it is (i) white, (ii) not black.

SOLUTION Total balls = 3 + 2 + 5 = 10.

(i) P(white) = 2/10 = 1/5 = 0.2

(ii) P(not black) = 1 − 5/10 = 5/10 = 1/2 = 0.5.

4. Two dice are rolled together. Find the probability that the sum of the numbers obtained is 7.

SOLUTION Total outcomes = 36.

Favourable outcomes for sum = 7: (1,6),(2,5),(3,4),(4,3),(5,2),(6,1) = 6

P(sum = 7) = 6/36

= 1/6.

Case-Based Questions

Case Study 1: School Canteen Survey

The school canteen manager wants to know the favourite snacks of students to plan the weekly menu. She surveys a random sample of 80 students. The results are: Samosa – 28 students, Sandwich – 20 students, Pasta – 16 students, Fruit Chaat – 16 students. There are 1200 students in the entire school.

a). What is the probability that a randomly chosen student from the sample prefers Samosa?

b). Estimate the number of students in the whole school who are likely to prefer Pasta.

c). Is this an example of experimental/statistical probability or theoretical probability? Justify your answer.

SOLUTION-

a) P(Samosa) = 28/80 = 0.35 or 35%.

b) Estimated Pasta lovers in school = (16/80) × 1200 = 0.2 × 1200 = 240 students.

c) This is statistical/experimental probability, since it is based on collected survey data from a sample, not on an assumption of equally likely outcomes

Case Study 2: The Snakes and Ladders Game

Riya is playing Snakes and Ladders with a fair 6-sided die. On her last four turns, she rolled a 6 every single time. Her friend Aman says, "You have rolled four 6's in a row — it is now very unlikely you will roll a 6 again, so a different number is 'due'."

a). What is the theoretical probability of rolling a 6 on Riya's next turn?

b). Is Aman's reasoning correct? Name the misconception he is demonstrating.

c). If Riya rolls the die 600 times, roughly how many times would you expect a 6 to appear, and which principle predicts this?

SOLUTION-

a) P(rolling a 6) = 1/6 ≈ 0.167, same as always for a fair die.

b) No, Aman's reasoning is incorrect. This is an example of the Gambler's Fallacy — the mistaken belief that past outcomes affect future independent events.

c) Roughly 100 times (1/6 × 600 ≈ 100). This is predicted by the Law of Large Numbers, which states that with more trials, experimental probability approaches theoretical probability

Higher Order Thinking Skills (HOTS) Questions

1. A game designer wants a spinner where the probability of winning a prize is exactly 0.3. If the spinner has 20 equal sectors, how many sectors should be marked 'Prize'? Explain your reasoning.

SOLUTION- P(Prize) = favourable/total

= x/20 = 0.3 → x = 6.

So 6 out of 20 sectors should be marked 'Prize'.

2. Two students each toss a coin 10 times. Student A gets 7 heads; Student B gets 4 heads. Does this mean Student A's coin is more likely to be biased than Student B's? Explain how the Law of Large Numbers would help you investigate this further.

SOLUTION- No — a small number of trials (10 tosses) can easily differ from the theoretical probability of 0.5 by chance alone; this does not indicate bias. The Law of Large Numbers suggests both students should toss their coins many more times (e.g., 500–1000 times) — if the relative frequency of heads still stays far from 0.5 after many trials, that would be better evidence of bias.

3. A factory tests 500 bulbs and finds 15 are defective. (i) Estimate the probability that a randomly chosen bulb from this factory is defective. (ii) If the factory produces 50,000 bulbs a month, estimate how many are likely to be defective. (iii) Explain one reason why this estimate might not be fully accurate for all future production.

SOLUTION- (i) P(defective) = 15/500 = 0.03 or 3%.

(iii) Estimated defective bulbs per month = 0.03 × 50,000 = 1500 bulbs.

(iii) This estimate assumes future production has the same defect rate as the tested sample; actual rates could vary due to changes in raw materials, machinery wear, or quality control over time.

PRACTISE QUESTIONS

MCQ

1. The minimum probability of an event is:

(a) 0 (b) 1 (c) 1/2 (d) −1.

2. The maximum probability of an event is:

(a) 0 (b) 1 (c) 1/2 (d) none of these.

3. What is the number of outcomes when a coin is tossed?

(a) 1 (b) 2 (c) 4 (d) 6.

4. What is the number of outcomes when a cubical dice is thrown?

(a) 2 (b) 4 (c) 6 (d) 3.

5. The sum of all the probabilities of all the possible outcomes of a trial is:

(a) 0 (b) 1 (c) −1 (d) 2.

6. If P(E) is the probability of an event E, then:

(a) $0 \le P(E) \le 1$ (b) $0 < P(E) < 1$ (c) $0 \le P(E) < 1$ (d) $0 < P(E) \le 1$.

7. $P(E) + P(\bar{E}) =$:(a) 0 (b) 1 (c) 2 (d) none of these

8. A coin is tossed 100 times with the following frequencies: Head: 75, Tail: 25. Find the probability of getting a head.

(a) 1/4 (b) 1/2 (c) 3/4 (d) 1.

9. The following table shows the birth day of 30 students of class X:

Day	Number of students
Sunday	3
Monday	6
Tuesday	2
Wednesday	1
Thursday	9
Friday	5
Saturday	4

Find the probability that a student was born on Saturday. Options:

(a) 2/15 (b) 1/6 (c) 1/3 (d) 1/2.

10. If A and B are the only two outcomes of an event and P(A) = 0.32, then value of P(B) would be:

(a) 0.38 (b) 0.68 (c) 0.78 (d) 0.32.

ASSERTION – REASON QUESTIONS

Questions number 11 and 12 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer from the codes below:

a)Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b)Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
c)Assertion (A) is true, but Reason (R) is false.
d)Assertion (A) is false, but Reason (R) is true.

11. Assertion (A): The probability of getting a 7 when a die is rolled is 1/6.

Reason (R): A die has 6 faces numbered 1 to 6.

12. Assertion (A): The probability of drawing a king from a deck of 52 cards is 1/13.

Reason (R): There are 4 king cards in a deck of 52 cards.

VERY SHORT QUESTIONS

13. Out of the past 250 consecutive days, its weather forecasts were correct 175 times. (i) What is the probability that on a given day it was correct? (ii) What is the probability that it was not correct on a given day?

14. On a particular day, the number of vehicles passing through a crossing is given below:

Vehicle	Frequency
Two-wheeler	57
Three-wheeler	33
Four-wheeler	30

15. The king, queen and jack of clubs are removed from a deck of 52 cards and then well shuffled. One card is selected at random from the remaining cards. Find the probability of getting (a) a heart (b) a king (c) the 10 of hearts.

16. In a school of 1250 students, 760 are boys and the remaining are girls. Find the probability that a student selected at random is a girl.

17. A coin is tossed 300 times and the following outcomes are recorded: Head: 165; Tail: 135. Determine the experimental probability of each outcome.

SHORT QUESTIONS

18.3. A die is rolled 25 times and outcomes are recorded as under:

Outcome	Frequency
1	4
2	5
3	5
4	6
5	1
6	0

It is thrown one more time. Find the probability of getting (a) an even number (b) a multiple of 3 (c) a prime number.

19.1. A survey of 500 families was conducted to know their opinion about a particular detergent powder. If 375 families liked the detergent powder and the remaining families disliked it, find the probability that a family chosen at random (i) likes the detergent powder (ii) does not like it.

20. Prem throws a pair of 6-sided dice. Write down an event that has a probability of 0 and an outcome that has a probability of 1.

LONG QUESTIONS

21. 30 plants were planted in each school out of 12 schools. After a month the number of plants that survived are given below:

School	Number of plants survived
1	22
2	15
3	12
4	24
5	27
6	10
7	13
8	22
9	17
10	9
11	20
12	25

What is the probability of survival of (i) more than 20 plants in a school? (ii) less than 10 plants in a school? (iii) exactly 22 plants in a school?

22.5. On a busy road, following data was observed about cars passing through it and number of occupants:

Number of occupants	Frequency
1	29
2	26
3	23
4	17
5	5

Suppose another car passes by. Find the chance that it has (i) exactly 5 occupants (ii) more than 2 occupants (iii) less than 5 occupants.

23.Cards marked with numbers 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from this box. Find the probability that the number on the card is (a) a number less than 14 (b) a number which is a perfect square (c) a prime number less than 20. [CBSE]

CASE STUDY QUESTIONS

24. A card is drawn at random from a well-shuffled deck of 52 cards.

A)Find the probability of drawing a king.
B)Find the probability of drawing a red card.
C)Find the probability of drawing a black face card.
25.A die is rolled twice.
A). Write the total number of possible outcomes.
B). Find the probability of getting a doublet (same number on both dice).
C). Find the probability of getting a sum greater than 10.

ANSWER KEY

1. (a)

2. (b)

3. (b)

4. (c)

5. (b)

6. (a)

7. (b)

8. (c)

9. (a)

10. (b)

11. (d)

12.(b)

13.Total number of days = 250. Number of days on which the weather forecast was correct = 175.

Probability that on a given day it was correct = 175/250 = 7/10.

Probability that it was not correct = 1 − 7/10 = 3/10.

14. Number of two-wheelers = 57, three-wheelers = 33, four-wheelers = 30. Total number of vehicles = 57 + 33 + 30 = 120.

Number of vehicles that are not four-wheelers = 57 + 33 = 90.

Probability = 90/120 = 3/4.

15. Total number of cards after removing king, queen and jack of clubs = 52 − 3 = 49.

(a) Number of hearts = 13; probability = 13/49.

(b) Number of kings remaining = 3; probability = 3/49.

(c) Number of 10 of hearts = 1; probability = 1/49.

16.P(G)= 49/125

17. P(H)=11/20

P(T)= 9/20

18. Total number of times a die is rolled = 25.

(a) Even numbers are 2, 4, 6. Probability = (5 + 6 + 0)/25 = 11/25.

(b) Multiples of 3 are 3 and 6. Probability = (5 + 0)/25 = 1/5.

(c) Prime numbers are 2, 3 and 5. Probability = (5 + 5 + 1)/25 = 11/25.

19.Total number of families = 500.

Families who like the detergent powder = 375.

Probability = 375/500 = 3/4.

Families who dislike it = 500 − 375 = 125.

Probability = 125/500 = 1/4.

20.(i) Event that has a probability of 0: The sum of the two numbers that appeared on the two dice is greater than 12.

(ii) Outcome that has a probability of 1: The sum of the two numbers that appeared on the two dice is 2 to 12.

21. Total number of schools = 12.

(i) More than 20 plants survived in 5 schools, so probability = 5/12.

(ii) Less than 10 plants survived in 1 school, so probability = 1/12.

(iii)Exactly 22 plants survived in 2 schools, so probability = 2/12 = 1/6.

22. Total number of subjects = 5.

(i) Subjects with at least 60% marks = 4,

so probability of first class = 4/5.

(ii) Subjects with 75% or above = 2,

so probability of distinction = 2/5.

(iii) Marks between 70% and 80% = 2

subjects (76% and 75% or the values in the table falling in the interval),

so probability = 2/5.

23.Total number of cards in the box = 100.

(a) Numbers less than 14 are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.

Their number = 12. Probability = 12/100 = 3/25.

(b) Perfect square numbers are 4, 9, 16, 25, 36, 49, 64, 81, 100.
Their number = 9. Probability = 9/100.

(c) Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
Their number = 8. Probability = 8/100 = 2/25.

24.(a) Kings = 4, Probability = 4/52 = 1/13.

(b) Red cards = 26, Probability = 26/52 = 1/2.

(c)Black face cards = 6, Probability = 6/52 = 3/26.

25.(a)Total outcomes = 6 × 6 = 36

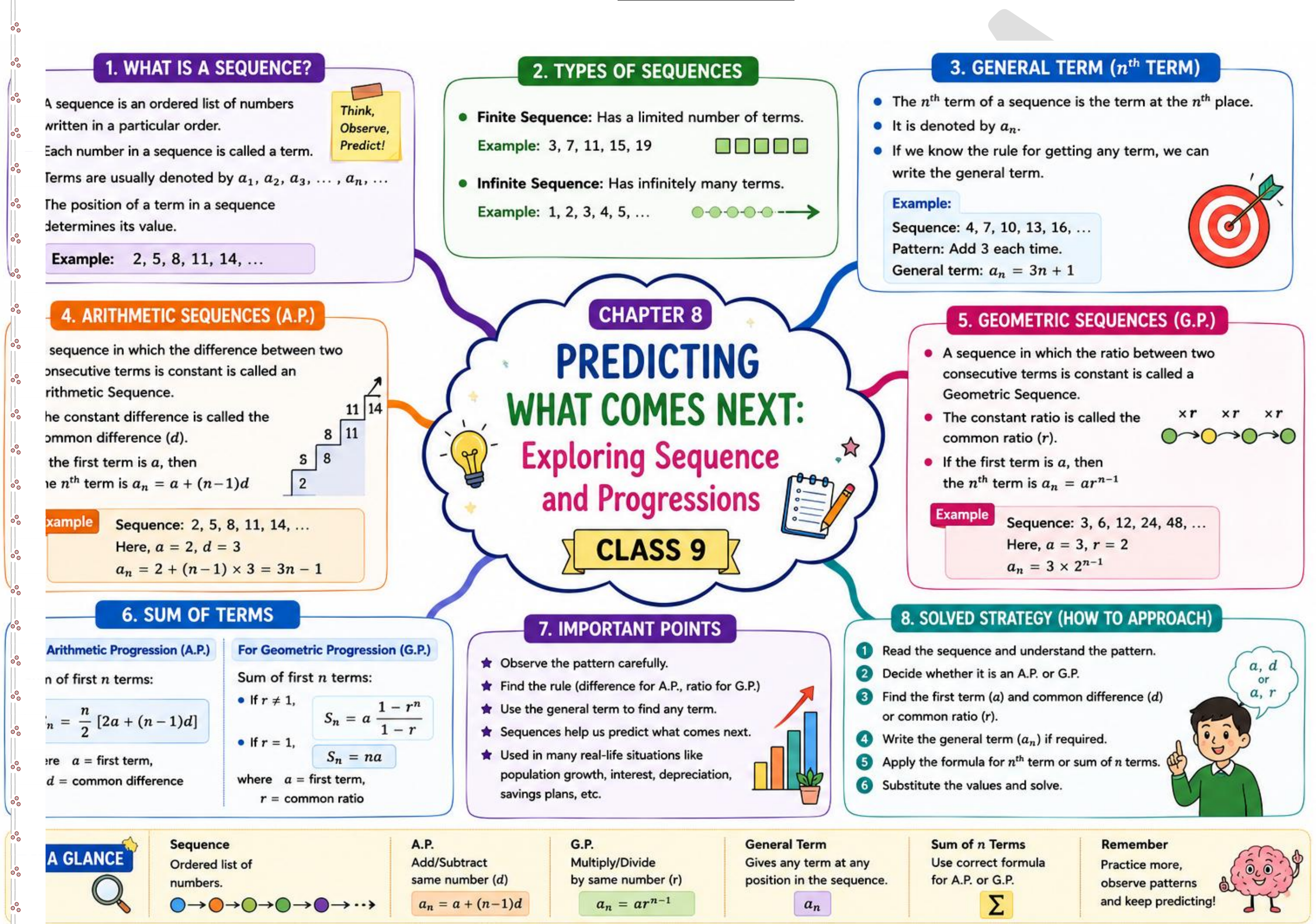
(b) Doublets = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)} → 6 outcomes → Probability = 6/36 = 1/6

(c)Sum > 10 → {(5,6), (6,5), (6,6), (4,6), (6,4)} and (5,6) etc. → 6 outcomes → Probability = 6/36 = 1/6

CHAPTER-8

Predicting What Comes Next: Exploring Sequences and Progressions

MIND MAP



Summary of chapter (Concepts and Formulae):-

Sequence: An ordered list of numbers where each number is called a term.

Notation: Terms are denoted using subscripts corresponding to their position in the sequence (t_1, t_2, t_3….. t_n). Sequences can also use different base letters, such as s_n or u_n

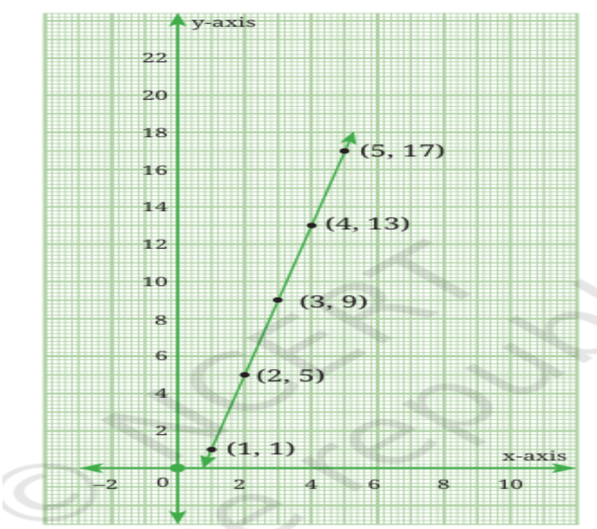
Types: Sequences can be finite (a limited number of terms) or infinite (continuing indefinitely)

Explicit Rule (Formula): A formula that expresses the n^{th} term directly in terms of its position number n. It enables direct calculation of any term without needing prior terms.

Recursive Rule (Formula): A formula that defines each term using one or more preceding terms alongside specified initial term(s).

Triangular Numbers: The sequence 1, 3, 6, 10, 15, ……., where each term represents the sum of all natural numbers up to its position.

Arithmetic Progression (AP): A sequence in which each term after the first is obtained by adding a constant value d (the common difference) to the previous term.Graphical Behaviour: Ordered pairs (n, t_n) of an AP form a straight line.



Growing pattern of squares represented by a linear pattern

Geometric Progression (GP): A sequence in which each term after the first is obtained by multiplying the previous term by a fixed non-zero constant r (the common ratio).

Graphical Behaviour: Ordered pairs (n, t_n) of a GP do not lie on a straight line.

Points emerging from a GP do not lie on a straight line

Fractals: Fractals are shapes or patterns that repeat themselves at different scales. This means that if you zoom in on a small part of a fractal, it looks similar to the whole! Fractals are found throughout nature — like in the branching of trees, in vegetables, such as cauliflower or broccoli, in the shape of snowflakes, or in the patterns of coastlines. They can be created using simple rules but can form very complex and beautiful designs. Fractals help us understand patterns in nature, and also lead us to important concepts in mathematics.

FORMULAE

Sum of First n Natural Numbers

$S_n = n(n + 1)/2$

Triangular Numbers (n^{th} term)

$T_n=\frac{n(n+1)}{2}$ $T_1= 1$, $t_n = t_{n-1} + n$ for $n >2$

Arithmetic Progression (AP)

$t_n = a + (n - 1)d$ $t_1 =a$, $t_n= t_{n-1} + d$ for $n >2$

(where a = first term, d =common difference)

Geometric Progression (GP)

$t_n = ar^{n-1}$ $t_1 =a$, $t_n = r\ t_{n-1}$ for $n >2$

Virahānka–Fibonacci sequence

A recursive rule or formula does not only have to involve the previous term — it could involve the previous two or more terms.

The most famous example of such a sequence is V_1, V_2,V_3 , … where , $V_1 = 1$, , $V_2 = 2$,

And $V_n = V_{n-1}+ V_{n-2}$ for $n \geq 3$.

We compute: $V_3 = V_2 + V_1 = 2 + 1 = 3$

$V_4 = V_3 + V_2 = 3 + 2 = 5$

$V_5 = V_4 + V_3 = 5 + 3 = 8$

So we get the sequence 1, 2, 3, 5, 8, 13, 21, 34, … where each term is obtained by adding the previous two.

It was first written down explicitly and studied by Virahānka in his work Vṛttajātisamuchaya in the 7th century CE. He discovered it in the context of Prakrit meter and poetry! It was further studied by the linguist-mathematicians Gopāla (c. 1135 CE) and Hemachandra (c. 1150 CE). The sequence was later also studied by the Italian mathematician Fibonacci (c. 1200 CE).

SOLVED EXAMPLES

1.What is the 10th term of the Arithmetic Progression 2, 7, 12, 17, ...?

A) 42 B) 47 C) 52 D) 57

Solution:First term $a = 2$, common difference $d = 7 - 2 = 5$.

Using the n-th term formula $a_n = a + (n - 1)d$:

$a_{10} = 2 + (10 - 1) \times 5 = 2 + 45 = 47$

Correct Option: B) 47

2.If the sum of the first n terms of an Arithmetic Progression is given by $S_n = 3n^2 + 5n$, what is the common difference d?

A) 3 B) 5 C) 6 D) 8

Solution:Find the first term a_1: $a_1 = S_1 = 3(1)^2 + 5(1) = 8$

Find S_2: $S_2 = 3(2)^2 + 5(2) = 12 + 10 = 22$

Find the second term a_2: $a_2 = S_2 - S_1 = 22 - 8 = 14$

Common difference $d = a_2 - a_1 = 14 - 8 = 6$.

Correct Option: C) 6

3. What is the 6th term of the Geometric Progression 3, 6, 12, 24, ...?

A) 48 B) 96 C) 192 D) 384

Solution:First term a = 3, common ratio r = 6 / 3 = 2.

Using the formula $a_n = a \times r^{n-1}$:

$a_6 = 3 \times 2^{6-1} = 3 \times 2^5 = 3 \times 32 = 96$

4.If a, b, c are three consecutive terms of a Geometric Progression, which of the following relations is correct?

A)b = (a + c) / 2 B) b^2 = ac C) b = ac D) 2b = a + c

Solution: By definition of GP, common ratio $r = b / a = c / b$.

Cross-multiplying gives $b^2 = ac$.

Correct Option: B) b^2 = ac

Question 5The sum to infinity of a Geometric Progression with first term a = 5 and common ratio r = 1/3 is:

A) 15 B) 15/2 C) 10 D) 5/2

Solution: Using the infinite sum formula $S\infty = a / (1 - r)$ for |r| < 1:

$S\infty$ = 5 / (1 - 1/3) = 5 / (2/3) = 15/2

Correct Option: B) 15/2

Short Answer Questions

1. Find the 15th term of the sequence defined by $a_n = (-1)^{n-1} \times n^2$.

Solution: Substitute $n = 15$ into the general term formula:

$a_{15} = (-1)^{15-1} \times (15)^2$

$a_{15} = (-1)^{14} \times 225 = 1 \times 225 = 225$

2. Which term of the AP 21, 18, 15, ... is -81?

Solution: Given $a = 21$, $d = 18 - 21 = -3$, and $a_n = -81$.

$a_n = a + (n - 1)d$

$-81 = 21 + (n - 1)(-3)$

$-102 = -3(n - 1)$

$n - 1 = 34 \Rightarrow n = 35$

The 35th term of the AP is -81.

3. Calculate the sum of the first 20 terms of the AP: 5, 9, 13, 17, ...

Solution: Here $a = 5$, $d = 9 - 5 = 4$, and $n = 20$.

Using the sum formula $S_n = (n / 2) \times [2a + (n - 1)d]$:

$S_{20} = (20 / 2) \times [2(5) + (20 - 1)4]$

$S_{20} = 10 \times [10 + 19(4)] = 10 \times [10 + 76] = 10 \times 86 = 860$

4. Find the 8th term of a Geometric Progression whose 3rd term is 12 and 6th term is 96.

Solution: Let a be the first term and r be the common ratio.

$a_3 = a \times r^2 = 12$ ....(1)

$a_6 = a \times r^5 = 96$ ....(2)

Divide equation (2) by equation (1):

$(a \times r^5) / (a \times r^2) = 96 / 12 \Rightarrow r^3 = 8 \Rightarrow r = 2$

Substitute $r = 2$ into equation (1):

$a \times (2)^2 = 12 \Rightarrow 4a = 12 \Rightarrow a = 3$

Now compute the 8th term:

$a_8 = a \times r^7 = 3 \times (2)^7 = 3 \times 128 = 384$

5. Insert two numbers between 3 and 81 so that the resulting four-term sequence forms a Geometric Progression.

Solution:Let the sequence be 3, G_1, G_2, 81.

First term $a = 3$, and fourth term $a_4 = 81$.

$a_4 = a \times r^3 \Rightarrow 81 = 3 \times r^3 \Rightarrow r^3 = 27 \Rightarrow r = 3$

Now evaluate G_1 and G_2:

$G_1 = a \times r = 3 \times 3 = 9$

$G_2 = a \times r^2 = 3 \times 3^2 = 27$

The two inserted geometric terms are 9 and 27.

Long Answer Type Questions

1. The 3rd term of an AP is 7 and the 7th term is 2 more than three times its 3rd term. Find the first term, the common difference, and the sum of the first 20 terms.

Solution: Let the first term be a and the common difference be d.

From the given conditions:

$a_3 = a + 2d = 7$ --- (1)

$a_7 = 3(a_3) + 2 \Rightarrow a + 6d = 3(7) + 2 = 23$ --- (2)

Subtract equation (1) from equation (2):

$(a + 6d) - (a + 2d) = 23 - 7$

$4d = 16 \Rightarrow d = 4$

Substitute $d = 4$ into equation (1):

$a + 2(4) = 7 \Rightarrow a + 8 = 7 \Rightarrow a = -1$

Calculate the sum of the first 20 terms (S_{20}):

$S_{20} = (20 / 2) \times [2a + (20 - 1)d]$

$S_{20} = 10 \times [2(-1) + 19(4)] = 10 \times [-2 + 76] = 10 \times 74 = 740$

First term $a = -1$, common difference $d = 4$, and $S_{20} = 740$. Answer

2. If the p^{th}, q^{th}, and r^{th} terms of an AP are x, y, and z respectively, prove that: $x(q - r) + y(r - p) + z(p - q) = 0$

Solution:Let A be the first term and D be the common difference of the AP.

$x = A + (p - 1)D$

$y = A + (q - 1)D$

$z = A + (r - 1)D$

Substitute x, y, and z into L.H.S. = $x(q - r) + y(r - p) + z(p - q)$:

$L.H.S. = [A + (p - 1)D](q - r) + [A + (q - 1)D](r - p) + [A + (r - 1)D](p - q)$

Group the terms by A and D:

$L.H.S. = A \times [(q - r) + (r - p) + (p - q)] + D \times [(p - 1)(q - r) + (q - 1)(r - p) + (r - 1)(p - q)]$

Evaluating coefficients:

Coefficient of A: $(q - r) + (r - p) + (p - q) = 0$

Coefficient of D: $(pq - pr - q + r) + (qr - qp - r + p) + (rp - rq - p + q) = 0$

Substituting back:

$L.H.S. = A(0) + D(0) = 0 = R.H.S.$

Hence proved.

3.Find the sum of n terms of the sequence: 7, 77, 777, 7777, ...

Solution: Let $S_n = 7 + 77 + 777 + \dots$ *to n terms*.

Factor out 7:

$S_n = 7 \times (1 + 11 + 111 + \dots$ *to n terms*$)$

Multiply and divide by 9:

$S_n = (7 / 9) \times (9 + 99 + 999 + \ldots \text{ to } n \text{ terms})$

Express each term as powers of 10:

$S_n = (7 / 9) \times [(10 - 1) + (10^2 - 1) + (10^3 - 1) + \ldots + (10^n - 1)]$

Group terms into two separate series:

$S_n = (7 / 9) \times [(10 + 10^2 + 10^3 + \ldots + 10^n) - (1 + 1 + 1 + \ldots n \text{ times})]$

The first series is a GP with first term $a = 10$, common ratio $r = 10$, and n terms:

$\text{GP Sum} = 10(10^n - 1) / (10 - 1) = 10(10^n - 1) / 9$

Substituting back into S_n:

$S_n = (7 / 9) \times [\, 10(10^n - 1) / 9 - n \,] = (70 / 81)(10^n - 1) - (7n / 9)$

4. The sum of three consecutive terms of a Geometric Progression is 56. If 1, 7, and 21 are subtracted from these numbers respectively, the resulting sequence forms an Arithmetic Progression. Find the three original numbers.

Solution: Let the three numbers in GP be a, ar, ar^2.

Sum condition:

$a + ar + ar^2 = 56 \Rightarrow a(1 + r + r^2) = 56$ --- (1)

Subtracting 1, 7, and 21 yields terms of an AP:

$(a - 1), (ar - 7), (ar^2 - 21)$

By AP property $2T_2 = T_1 + T_3$:

$2(ar - 7) = (a - 1) + (ar^2 - 21)$

$2ar - 14 = a + ar^2 - 22$

$a - 2ar + ar^2 = 8 \Rightarrow a(1 - 2r + r^2) = 8$ --- (2)

Divide equation (1) by equation (2):

$[a(1 + r + r^2)] / [a(1 - 2r + r^2)] = 56 / 8$

$(1 + r + r^2) / (1 - 2r + r^2) = 7$

$1 + r + r^2 = 7(1 - 2r + r^2) \Rightarrow 1 + r + r^2 = 7 - 14r + 7r^2$

$6r^2 - 15r + 6 = 0 \Rightarrow 2r^2 - 5r + 2 = 0$

Factorize the quadratic equation:

$(2r - 1)(r - 2) = 0 \Rightarrow r = 2$ or $r = 1/2$

Case 1 (r = 2):

From (1): $a(1 + 2 + 4) = 56 \Rightarrow 7a = 56 \Rightarrow a = 8$.

The numbers are 8, 16, 32.

Case 2 (r = 1/2):

From (1): $a(1 + 1/2 + 1/4) = 56 \Rightarrow a(7/4) = 56 \Rightarrow a = 32$.

The numbers are 32, 16, 8.

The three numbers are 8, 16, and 32.

5. The ratio of the sum of m terms and n terms of an AP is $m^2 : n^2$. Show that the ratio of its m-th term and n-th term is $(2m - 1) : (2n - 1)$.

Solution: Let a be the first term and d be the common difference.

Given:

$S_m / S_n = [(m/2) \times (2a + (m-1)d)] / [(n/2) \times (2a + (n-1)d)] = m^2 / n^2$

Simplify:

$[2a + (m-1)d] / [2a + (n-1)d] = m / n$

Cross-multiply:

$n \times [2a + (m-1)d] = m \times [2a + (n-1)d]$

$2an + n(m-1)d = 2am + m(n-1)d$

$2an - 2am = m(n-1)d - n(m-1)d$

$2a(n - m) = d(mn - m - mn + n) = d(n - m)$

Divide both sides by $(n - m)$:

$2a = d$

Now compute the ratio of the m-th term (a_m) to the n-th term (a_n):

$a_m / a_n = [a + (m-1)d] / [a + (n-1)d]$

Substitute $d = 2a$:

$a_m / a_n = [a + (m-1)(2a)] / [a + (n-1)(2a)] = a(1 + 2m - 2) / a(1 + 2n - 2) = (2m - 1) / (2n - 1)$

Thus, the ratio of the m-th term to the n-th term is (2m - 1) : (2n - 1).

Case Study 1: Bouncing Ball

An elastic ball is dropped from a height of 100 meters onto a hard floor. Each time it strikes the floor, it bounces back to a height equal to 1/2 of the height from which it fell.

Q1. Write the heights reached by the ball after the 1st, 2nd, and 3rd bounces.

Solution: Height after 1st bounce $h_1 = 100 \times (1/2) = 50$ m

Height after 2nd bounce $h_2 = 50 \times (1/2) = 25$ m

Height after 3rd bounce $h_3 = 25 \times (1/2) = 12.5$ m

Q2. Express the height h_n reached after the n-th bounce as a function of n.

Solution: The sequence of heights 50, 25, 12.5, ... forms a Geometric Progression with first term a = 50 and common ratio r = 1/2.

$$h_n = 100 \times (1/2)^n \text{ meters}$$

Q3. Calculate the total vertical distance traveled by the ball before it comes to rest.

Solution: Total vertical distance D includes the initial fall plus twice the height of every bounce (up and down):

$$D = 100 + 2 \times (h_1 + h_2 + h_3 + ...)$$

$$D = 100 + 2 \times (50 + 25 + 12.5 + ...)$$

The inner series is an infinite GP with a = 50 and r = 1/2:

$$S_\infty = a / (1 - r) = 50 / (1 - 1/2) = 100 \text{ m}$$

Substituting this back:

$$D = 100 + 2(100) = 300 \text{ meters}$$

Case Study 2: Personal Monthly Savings Plan

An employee decides to save money every month starting in January. In the 1st month, she saves ₹ 1,000. In each subsequent month, she increases her monthly savings by ₹ 200 over the previous month.

Month	Savings (₹)
January (n = 1)	1,000
February (n = 2)	1,200
March (n = 3)	1,400

Q1. How much money will she save in the 12th month (December)?

Solution: Monthly savings form an AP with a = 1000 and d = 200.

$a_{12} = a + (12 - 1)d = 1000 + 11(200) = 1000 + 2200 = ₹\ 3,200$

Q2. What will be her total cumulative savings at the end of 1 year (12 months)?

Solution: Calculate S_{12}:

$S_{12} = (12 / 2) \times [2a + (12 - 1)d] = 6 \times [2(1000) + 11(200)] = 6 \times [2000 + 2200] = 6 \times 4200 = ₹\ 25,200$

Q3. In which month will her monthly savings reach ₹ 3,000, and what will her total cumulative savings be up to that month?

Solution: Set $a_n = 3000$:

$1000 + (n - 1)200 = 3000 \Rightarrow (n - 1)200 = 2000 \Rightarrow n - 1 = 10 \Rightarrow n = 11$

Her monthly savings will reach ₹ 3,000 in the 11th month (November).

Cumulative savings up to the 11th month (S_{11}):

$S_{11} = (11 / 2) \times [2(1000) + (11 - 1)200] = (11 / 2) \times [2000 + 2000] = (11 / 2) \times 4000 = ₹\ 22,000$

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Questions for Practice

MCQ

1. A sequence is:

(a) a random collection of numbers. (b) an ordered list of numbers

(c) a graph (d) a table

2. A recursive rule defines:

(a) only the nth term (b) first term and relation with previous term

(c) only common (d) only sum

3. Write the general term of the sequence :- 4 , 7 , 10 , 13 , ……..

(a) 3n+1 (b) n+3 (c) 7n+1 (d) n+4

4. If p-1 , p+3 ,3p-1 are in AP, then p is equal to

(a) 2 (b) 3 (c) 4 (d) 5

5. In an AP, if the common difference (d) = - 4 and the seventh term (a_7) is 4, then the first term is

(a) 32 (b) 28 (c) 24 (d) 20

6. In the progression 65, 61, 57 ... first negative term is which term?

(a) 18 (b) 19 (c) 20 (d) 17

7. Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then, the difference betweentheir 4th terms is

(a) 9 (b) 1 (c) 8 (d) 7

8. The 9th term of the GP 2, 2 /2, 4, 82, ... is

(a) 64 (b) 16 (c) 128 (d) 32

9. Which term of the sequence 3, 6, 12 , ………. is 768?

(a) 6 (b) 10 (c) 8 (d) 9

10.The third term of a geometric progression is 4.The product of the first five terms is

(a) 4^5 (b) 4^6 (c) 4^3 (d) 4^4

11.In Stage n of the Sierpiński triangle, if Stage 0 has 1 black triangle, how many black triangles are there at Stage n?

(a) 2^n (b) 3^n (c) 4^n (d) 3^{n-1}

12. If the initial area of Stage 0 of the Sierpiński triangle is 1 square unit, what is the area of the black region at Stage n?

(a) $(1/2)^n$ b) $(3/4)^n$ c) $(1/4)^n$ d) 1 - n/4

13.What is the sum of all 2-digit numbers divisible by 3?

a) 1655 b) 1665 c) 1565 d) 1680

14. In a Geometric Progression a, ar, ar^2, ..., the formula for the n-th term is:

a) ar^n b) ar^{n-1}c) $a + r^{n-1}$d) $(ar)^{n-1}$

15.If $s_n = 5n - 2$, why is 471 not a term of this sequence?

a) because n = 94 gives 468 b) because solving 5n - 2 = 471 gives n = 94.6

c) because 471 is odd d) because sequence is decreasing

16. The recursive formula for the AP 1, 5, 9, 13, ... is:

a) $t_1 = 1, t_n = t_{n-1} + 4$ b) $t_1 = 1, t_n = 4t_{n-1}$

c) $t_1 = 5, t_n = t_{n-1} + 1$ d) $t_1 = 1, t_n = t_{n-1} + 3$

17.In a fractal, the number of elements triples at each step. If the first step has 4 elements then the number of elements in the 4th step

(a) 87 (b) 108 (c) 99 (d) 101

18. In a square fractal pattern, the number ofsmaller squares formed at each step follows a GP with first term 1 and common ratio 8.The number of squares in the 5th step is

(a) 512 (b) 4096 (c) 32768 (d) 64

Directions:

For each question, two statements are given—one labelled Assertion (A) and the other labelled Reason (R). Select the correct option:

(A) Both (A) and (R) are true, and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true, but (R) is NOT the correct explanation of (A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

19. Assertion (A): 0 is the 8th term of the progression 21, 18, 15, ...

Reason (R): The common difference of the sequence 21, 18, 15, ... is 3.

20. Assertion (A): For the sequence defined by $S_n = 5n - 2$, the number 471 is a valid term.

Reason (R): A number belongs to a sequence if and only if its calculated term position n is a positive integer

Very Short Answer Questions

21. Find the first four terms of the sequence given by the recursive rule $u_1 = 1$, $u_n = 2u_{n-1} + 3$ for $n \geq 2$.

22. Find the explicit formula for the n-th term of the GP: 5, 25, 125, ...

23. Consider the sequence given by the explicit rule $t_n = 3n - 7$. (i) Find its 12th term. (ii) Which term of the sequence is 332?

24. An AP consists of 50 terms in which the 3rd term is 12 and the last term is 106. Find the 29th term.

25. Determine whether 0 is a term of the AP: 21, 18, 15, ...

Short Answer Questions

26. Derive algebraically the formula for the sum of the first n natural numbers $S_n = 1 + 2 + 3 + ... + n$.

27. Find the 31st term of an AP whose 11th term is 38 and 16th term is 73.

28. The sum of the first three terms of a GP is 26, and the sum of their squares is 364. Find the terms of the GP.

29.If the first term of the and n^{th} term of a G.P. are a and b respectively and I P is the product of first n terms , then prove that $P^2 =(ab)^n$.

30. The 4^{th} ,6^{th} and the last term of a G.P. are 10,40,and 640 respectively. If the common ratio is positive , Find the first term , common ratio and the number of terms of the series.

31. How many multiples of 4 lies between 10 and 250.

Long Answer Questions

32.The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of the 2^{nd} hour, 4^{th} hour and n^{th} hour?

33.A person books a taxi charging a fixed booking fee of ₹200 plus ₹40 per kilometer travelled.

(i) Write the sequence of total fares after travelling 1 km, 2 km, 3 km, etc.

(ii) Derive the explicit formula for the n-th term.

(iii) Calculate total fare for 10 km.

34. Which term of the A. P. -7,-12, -17,-22 ….. will be -82. Is -100 any term of the A.P. ? Give reason for your answer.

35. A ball is dropped from a height of 80 meters. After hitting the ground, it bounces back to 60% of the height from which it fell. It continues bouncing in this way.

(i) Write the explicit formula for the height after n-th bounce.

(ii) What height does it reach after 5th bounce?

Case Study Based Questions

36. India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. Theproduction of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

Based on the above information, answer the following questions:

(i) Find the production during first year.

(ii) Find the production during 8th year.

(iii) (a) In which year, the production is 29,200.

OR

(b) Find the difference of the production during7th year and 4th year.

37. A school has decided to plant some endangered trees on 51st World Environment Day in the nearest park. They have decided to plant those trees in few concentric circular rows such that each succeeding row has 20 more trees than the previous one. The first circular row has 50 trees.

Based on the above given information, answer the following questions.

(i) How many trees will be planted in the 10^{th}row ?

(ii) How many more trees will be planted in the 8th row than in the 5th row ?

(iii) (a) If the number of trees planted in the 5^{th} row is 130 and in the 15th row is 330 and the plantation follows on arithmetic progression (AP), find the number of trees planted in the 31st row.

Or

(b) If the number of trees planted in the 4^{th} row is 110 and in the 12th row 270, and the plantation follows an arithmetic progression (AP), find the number of trees planted in 25th row.

38. A sequence of non-zero numbers is said to be a geometric progression, if the ratio of each term except the first one by its preceding term is always constant. Rahul being a plant lover decides to open a nursery and he bought few plants with pots.

He wants to place pots in such a way that number of pots in first row is 2, in second row is 4 and in third row is 8 and so on.

Based on the above information, answer the following questions.

(i) Find the constant multiple by which the number of pots is increasing in every row.

(ii) Find the number of pots in 8th row.

(iii) (a) What is the difference in number of pots placed in 7th row and 5th row?

Or

(b) What is the product in number of pats placed in 5th, 6th and 9th row?

==

Answers

1.(b) an ordered list of numbers

2.(b) first term and relation with previous term

3. (a) $3n + 1$

4. (c) 4

5. (b) 28

6. (a) 18

7. (d) 7

8. (d) 32

9. (d) 9

10. (c) 4^5 (or 1024)

11. (b) 3^n

12. (b) $(3/4)^n$

13. (b) 1665

14. b) $a\ r^{(n-1)}$

15. b) because solving $5n - 2 = 471$ gives $n = 94.6$ (not an integer)

16. a) $t_1 = 1$, $t_n = t_{(n-1)} + 4$

17 (b) 108

18 (b) 4096

19. (C) (A) is true, but (R) is false.

20. (D) (A) is false, but (R) is true

Very Short Answer Questions

21. $u_1 = 1$

$u_2 = 2(1) + 3 = 5$

$u_3 = 2(5) + 3 = 13$

$u_4 = 2(13) + 3 = 29$

First four terms: 1, 5, 13, 29

22. First term $a = 5$, common ratio $r = 5$.

Explicit formula: $t_n = 5x\ 5^{(n-1)} = 5^n$

23. (i) $t_{12} = 3(12) - 7 = 36 - 7 = 29$

(ii) $3n - 7 = 332 \Rightarrow 3n = 339 \Rightarrow n = 113$. So, 332 is the 113th term.

24. $a + 2d = 12$ ... (1)

$a + 49d = 106$ ... (2)

Subtracting (1) from (2): $47d = 94 \Rightarrow d = 2$.

$a = 12 - 4 = 8$.

29th term $a_{29} = 8 + 28(2) = 64$.

25. $a = 21$, $d = -3$.

Set $a_n = 0 \Rightarrow 21 + (n - 1)(-3) = 0 \Rightarrow 21 = 3(n - 1) \Rightarrow n - 1 = 7 \Rightarrow n = 8$.

Yes, 0 is the 8th term of the AP.

Short Answer Questions

26. Let $S_n = 1 + 2 + 3 + ... + n$ --- (1)

Reversing the order: $S_n = n + (n - 1) + (n - 2) + ... + 1$ --- (2)

Adding (1) and (2) term-by-term:

$2\ S_n = (n + 1) + (n + 1) + ... + (n + 1)$ [n times]

$2\ S_n = n(n + 1) \Rightarrow S_n = n(n + 1) / 2$.

27. $a + 10d = 38$ ... (1)

$a + 15d = 73$ ... (2)

Subtracting (1) from (2): $5d = 35 \Rightarrow d = 7$.

From (1): $a + 70 = 38 \Rightarrow a = -32$.

31st term $a_{31} = -32 + 30(7) = -32 + 210 = 178$.

28 A Let the terms be a, ar, ar^2.

$a(1 + r + r^2) = 26$ --- (1)

$a^2(1 + r^2 + r^4) = 364$ --- (2)

Squaring (1): $a^2 (1 + r + r^2)2 = 676$ --- (3)

Dividing (3) by (2): $(1 + r + r^2)^2 / (1 + r^2 + r^4) = 676 / 364 = 13 / 7$.

Since $(1 + r^2 + r^4) = (1 + r + r^2)(1 - r + r^2)$, we get:

$(1 + r + r^2) / (1 - r + r^2) = 13 / 7$

$7 + 7r + 7r^2 = 13 - 13r + 13^2 \Rightarrow 6^2 - 20r + 6 = 0 \Rightarrow 3^2 - 10r + 3 = 0$.

Solving yields r = 3 or r = 1/3.

If r = 3: a(1 + 3 + 9) = 26 => 13a = 26 => a = 2. Terms are: 2, 6, 18.

If r = 1/3: Terms are: 18, 6, 2.

29. First term = a, n-th term b = a $r^{(n-1).}$

$P = ax(ar)x(ar^2) \text{ x} \ldots x(ar^{(n-1)}) = a^n \text{ x } r^{(0 + 1 + 2 + \ldots + n-1)}$

Sum of exponents of r = (n-1)n / 2.

So, $P = a^n x r^{(n(n-1)/2).}$

$P^2 = a^{(2n)} x r^{(n(n-1))} = [a^2 \text{ x } r^{(n-1)})]^n = [ax(a\, r^{(n-1))}]^n = (ab)^n$.

Hence proved.

30.$ar^3 = 10$ ... (1)

$ar^5 = 40$ ... (2)

Dividing (2) by (1): $r^2 = 4 \Rightarrow r = 2$ (since r > 0).

From (1): a(8) = 10 => a = 5/4 = 1.25.

Last term = $a\, r^{(n-1)} = 640 \Rightarrow (5/4)\, x2^{(n-1)} = 640 \Rightarrow 2^{(n-1)} = 512 = 2^9 \Rightarrow n - 1 = 9 \Rightarrow n = 10$.

First term = 5/4, Common ratio = 2, Number of terms = 10.

31. Multiples of 4 are: 12, 16, 20, ..., 248.

First term a = 12, d = 4, last term $a_n = 248$.

248 = 12 + (n - 1)4 => 236 = 4(n - 1) => n - 1 = 59 => n = 60.

There are 60 multiples of 4.

Long Answer Questions

32. Initial count a = 30. Common ratio r = 2.

Number of bacteria at end of hour t = ar^t.

(i) At end of 2nd hour = $30x2^2$ = 120

(ii) At end of 4th hour = $30x2^4$ = 480

(iii) At end of nth hour = $30x2^n$

33. (i) Fare after 1 km = 200 + 40 = ₹240

Fare after 2 km = 200 + 40(2) = ₹280

Fare after 3 km = 200 + 40(3) = ₹320

Sequence: ₹240, ₹280, ₹320, ...

(ii) Explicit formula for nth term: $t_n = 200 + 40n$

(iii) Total fare for 10 km: t_{10} = 200 + 40(10) = ₹600.

34. a = -7, d = -5.

(i) Set a_n = -82 => -7 + (n - 1)(-5) = -82 => -5(n - 1) = -75 => n - 1 = 15 => n = 16.

So -82 is the 16th term.

(ii) Set a_n = -100 => -7 + (n - 1)(-5) = -100 => -5(n - 1) = -93 => n - 1 = 18.6 => n = 19.6.

Since n is not a natural number (positive integer), -100 is NOT a term of this A.P.

35.(i) Initial height = 80 m, multiplier r = 0.6.

Height after n-th bounce: $h_n = 80 \times (0.6)^n$ meters.

(ii) Height after 5th bounce: $h_5 = 80x (0.6)^5$ = 80x0.07776 = 6.2208 meters.

Case Study Based Questions

36 a + 5d = 16000 ... (1)

a + 8d = 22600 ... (2)

Subtracting (1) from (2): 3d = 6600 => d = 2200.

From (1): a = 16000 - 5(2200) = 5000.

(i) First year production = 5,000 sets.

(ii) 8th year production $a_8 = 5000 + 7(2200) = 20,400$ sets.

(iii) (a) $5000 + (n - 1)2200 = 29200 \Rightarrow 2200(n - 1) = 24200 \Rightarrow n - 1 = 11 \Rightarrow n = 12$.

So production reaches 29,200 in the 12th year.

OR

(b) Difference = $a_7 - a_4 = 3d = 3(2200) = 6,600$ sets.

37 Answer: Base info: First row $a = 50$, $d = 20$.

(i) 10th row $a_{10} = 50 + 9(20) = 230$ trees.

(ii) Difference = $a_8 - a_5 = 3d = 3(20) = 60$ more trees.

(iii) (a) $a + 4d = 130$, $a + 14d = 330 \Rightarrow 10d = 200 \Rightarrow d = 20$, $a = 50$.

31st row $a_{31} = 50 + 30(20) = 650$ trees.

OR

(b) $a + 3d = 110$, $a + 11d = 270 \Rightarrow 8d = 160 \Rightarrow d = 20$, $a = 50$.

25th row $a_{25} = 50 + 24(20) = 530$ trees.

38. (i) Given sequence of pots in rows: 2, 4, 8, ...

Common Ratio (r) = (Second term) / (First term) = 4 / 2 = 2

Thus, the constant multiple is 2.

(ii) Formula for n-th term of GP: $a_n = a \cdot r^{(n-1)}$

First term (a) = 2, Common ratio (r) = 2, and Row number (n) = 8

$a_8 = 2 \times 2^{(8-1)} = 2 \times 2^7$

$a_8 = 2 \times 128 = 256$

256 pots Answer

(iii - (a))

Number of pots in 7th row (a_7) = $2 \times 2^{(7-1)} = 2 \times 2^6 = 2 \times 64 = 128$

Number of pots in 5th row (a_5) = $2 \times 2^{(5-1)} = 2 \times 2^4 = 2 \times 16 = 32$

Difference = $a_7 - a_5 = 128 - 32 = 96$

96 pots Answer

OR

(iii - (b))

Pots in 5th row (a_5) = $2 \times 2^4 = 32$ (or 2^5)

Pots in 6th row (a_6) = $2 \times 2^5 = 64$ (or 2^6)

Pots in 9th row (a_9) = $2 \times 2^8 = 512$ (or 2^9)

Product = $a_5 \times a_6 \times a_9 = 2^5 \times 2^6 \times 2^9 = 2^{(5+6+9)} = 2^{20}$ Answer

=================END===============